DECISION SUPPORT FOR ATTACK SUBMARINE COMMANDERS

TECHNICAL REPORT

80-11

By Marvin S. Cohen & Rex V. Brown

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Decision Science Consortium, Inc.
7700 Leesburg Pike, Suite 421
Falls Church, Virginia 22043
(703) 790-0510

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SUMMARY

In work funded by the Engineering Psychology Program of the Office of Naval Research, Decision Science Consortium, Inc. (DSC) has explored the application of decision aids to attack submarine command and control. Analysis of decision requirements and current practice within various scenarios has led to consideration of three broad classes of aids:

- Inference aids, which assist in establishing probabilities for critical states of affairs (e.g., target classification and range),
- Alerting aids, which notify appropriate personnel when a critical threshold selected by them is exceeded by some indicator (e.g., the probability of being within counter-detection range),
- Prompting aids, which suggest and prioritize possible courses of action (e.g., approach maneuvers, weapon selection, time and method for communication, torpedo evasion maneuvers) given the inputs and objectives of the Commanding Officer (CO).

One context was singled out for detailed attention--passive target ranging with the intent to engage an enemy. Work on target ranging has typically treated it as a measurement problem, with improvement coming through new sensor systems or automated ranging techniques. DSC's approach is complementary, with a focus on the total decision-making context. Unless he is already under attack, the commanding officer decides to launch a weapon only when he is reasonably sure that the target is within weapon range and that the uncertainty in target localization is within the search capability of the weapon. However, in order to assess target range, he must select informally from numerous inconsistent solutions; and his assessment of uncertainty is not systematically aided.

An attack may be unnecessarily delayed because he is unable to exploit all the available information on target range in a timely manner.

Three kinds of aids have been developed on a conceptual level for this situation:

(i) For each solution technique, a probabilistic range assessment is provided which takes explicit account of variable and fixed sources of error. Error in a particular solution is decomposed into its contributing sources by a technique ("Decomposed Error Analysis") developed by Dr. Rex Brown (1969). Assessments of these components may be based on prior research (e.g., comparison of actual and estimated values during exercises) or may be adjusted on the spot. Objective and subjective information are accomodated and synthesized in a systematic way.

(ii) The results of the separate passive ranging techniques are pooled to produce a single probabilistic range assessment. The method being developed takes account both of the (shifting) relative validity of the different techniques and the degree of overlap or redundancy in their sources of data (Brown and Lindley, 1978; Lindley, Tversky, and Brown, 1979; Freeling, 1980). The output reflects in a readily understood way all the available sources of information on target range.

(iii) The resultant range assessment is used to alert the CO to critical dangers or opportunities: e.g., when the probability that the target is within weapon range exceeds a preset threshold.

The proposed aids are not intended to be "black boxes". At each level, inputs and results of processing are subject to adjustment or override by the CO or appropriate members of his staff. The aids are designed to support and supplement human judgment without displacing it. They are able to systematically combine objective and subjective sources of information. Thus, they will enhance, rather than diminish, the CO's control of the ship.

The feasibility of objective estimation of parameters for these aids has been demonstrated by reference to Rangex data.

In follow-on research, DSC will seek, first, to demonstrate the quantitative validity of the aids already proposed; second, to develop an action-prompting aid in the same passive approach context; and finally, to continue its study of submarine decision-making contexts in order to determine decision aid requirements.

1.0 INTRODUCTION

1.1 The Problem

A nuclear attack submarine must be capable of gathering information about its enemy while employing methods and sources of data which are severely constrained. Such methods must provide to the enemy as little information as possible about the ship that uses them, including even its presence. In particular, assessing the distance of a target from one's own ship while remaining undetected is a critical task if one's mission is to engage hostile contacts or to perform surveillance.

1.2 Current Approaches

Typically, target ranging has been conceptualized as a measurement problem. Two rather distinct lines of effort have flowed from that conceptualization. One line is concerned with the design and improvement of sensor systems. The other line has sought new algorithms and software implementations for estimating target range from sensor inputs. It is undeniable that there have been impressive advances in both areas. New sources of data have become available (e.g., sophisticated electronic countermeasures and new processes of sonar detection) which are effective at very long ranges. At the same time automatic and interactive target ranging techniques within the fire control system have taken a place beside the manual methods.

On the other hand, shortcomings in this approach have also become apparent. Every new advance produces an additional "black box" whose workings and output are seldom fully understood by its users, and which must somehow function in harmony with numerous other, independently developed devices. The result is that the officer responsible for an engagement is inundated with unselected and undigested information. Much of this may not be relevant to the problem at hand. Conversely, highly pertinent information may go unnoted.

1.3 Decision-Oriented Approach

What we have developed is a complementary approach. It should be clear that target range assessment takes place in a decision-making context. The Commanding Officer (CO) in a battle situation must decide when to fire, where, and with what weapons. The objective of target range assessment is not to grind the accuracy of localization to as fine a point as possible, but to serve the functions of combat (or surveillance, etc.). Technical advances in sensor-guided weaponry have in fact dramatically reduced the need for precision in target localization on-board the submarine. Thus the benefits of information gathering should be continually weighed against its costs, i.e., possible counterdetection (followed by evasion or attack). By the same token, there is a premium on making the best use of the information already available at any given time.

At the system design level, the proliferation of specialized subsystems and techniques must be balanced and guided by consideration of combat functions. Such a top-down analysis cannot ignore the users. Overall system design should focus on the actual impact which information is expected to have on judgment and decision-making, given constraints of time and cognitive capacity. Such a higher order system would make information available to the command staff when it is needed and in the form it is needed to improve decisions.

1.4 Room for Improvement in Target Ranging

What then are the real needs of a commanding officer in the target ranging situation? Experienced submarine officers have tended to reiterate, in conversations with us, points that are also made in various publications. Three major themes have emerged:

(1) The CO lacks an adequate assessment of the degree of confidence he should place in a ranging solution. He may be unable, therefore, to make a well-founded choice between continued data collection and analysis versus immediate attack. In exercises, target range estimates at time of fire are typically more accurate than they need to be. The tactical flexibility of the Mark 48 torpedo is thus not being exploited. Moreover, in order to get a better feel for the quality of a solution, the CO is tempted to become immersed in the details of a particular analytical procedure. In doing so, he loses his perspective on the total situation and wastes the time and attention he needs to make higher level judgments regarding, for example, approach maneuvers and the timing of the attack.

(2) Even if an assessment of solution quality were available to the CO, time of fire may be unnecessarily delayed if solution quality is not maximized. Several procedures are available for estimating target range. However, each is characterized by significant uncertainty, and no one of them alone exhausts the relevant evidence.

In these circumstances, uncertainty can be reduced by taking systematic account of the results of all procedures. In the absence of a procedure for doing so, the CO tends to base decisions regarding target range on a single estimation technique. In doing so, he not only ignores other methods which may, on a given occasion, provide better information. By relying on a single method (even if it is the best), he takes into account only a fraction of the available data.

(3) Finally, even if the best estimate of target range had been extracted from all available data, together with an accurate assessment of its precision, there is a feeling that such information might not be utilized in an optimal manner. The CO must combine available knowledge about a target's range, course, and capabilities, knowledge of his own weapon's capability, the value of destroying the target, and his own attitudes toward risk in order to decide when to launch an attack. The stakes contingent on a proper integration of these factors are very high. An ill-timed attack can increase the chances of target evasion or own ship destruction.

1.5 Personalist Decision Aids

The commanding officer's problems would not, of course, be solved by devices which simply automated each of these functions. Such devices might well be ignored--and would certainly not be trusted. Each could become another black box, in which case the CO would be at a loss--once again--to assess its credibility and integrate its output with other considerations.

Moreover, there would surely be valid reasons for mistrust. The large number of factors which enter into an attack decision, or even into an assessment of target range, cannot be fully anticipated and programmed in advance. Some factors cannot be objectively measured in any case (e.g., the value of the target). On the other hand, experienced submariners are said to acquire (despite the problems mentioned above) an almost instinctive ability to size up a situation and act appropriately.

Improvement in target ranging must come, therefore, from aids which support and supplement judgment without displacing it. Such aids, which we refer to as "personalist", will allow the CO to interpose his own assessments in addition to or in place of sensor data and prior research, at any stage of processing. But they will rapidly and systematically integrate subjective inputs with the objective data which is retained. Confidence in the output of such an aid will be based on a thorough understanding of and control over its inputs.

1.6 Completed Research

In work funded by the Engineering Psychology Program of the Office of Naval Research and described in this report, DSC has explored the application of decision aids to submarine command and control. The project has confined itself to the undersea portions of missions on board nuclear attack submarines.

This research, constituting one year of effort, has involved three major phases: identification of aid requirements in a variety of scenarios, development of specific technical concepts for aids in the target ranging situation, and demonstrations of the feasibility of quantifying the proposed aids. They are discussed in the following two chapters and Appendix E, respectively. Appendix A amplifies the identification of aid requirements, and Appendices B through D expand on technical aspects of the aids.

In the target ranging situation, DSC has outlined concepts for personalist decision aids which are responsive to the problems of assessing confidence, pooling range solutions, and alerting to critical ranges which form the basis for decisions about action.

Throughout this project a critical role has been played by feedback and advice from individuals with command-level Fleet experience. Opportunities to observe training exercises, on video tape and through personal visits to the Naval Submarine School, have also proven quite valuable. Appendix F summarizes this activity.

2.0 IDENTIFICATION OF AID REQUIREMENTS

2.1 Review of Submarine Setting

A general review of submarine decision contexts was undertaken in conjunction with experienced submarine command personnel, researchers at NUSC and elsewhere, and by examination of relevant Naval publications. Appendix A summarizes this work.

The identification of aid requirements within a scenario was of necessity begun in an informal manner--making use of the educated judgments of those most directly familiar with the problems. In tandem with this informal approach, however, and building upon it, an effort has been made to systematize the mapping of decision contexts onto decision aids. The methodology of taxonomy matching (Brown and Ulvila, 1977) involves the identification of characteristics of decision contexts which generally call for certain types of aid and for the formulation of general matching principles.

2.2 Selection of Promising Situations and Aids

As a result of these efforts, a subset of the decision situations were selected which were considered promising candidates for aids, and possible functions of aids in the selected situations were proposed.

Three broad classes of aids were considered:

- Inference aids, which assist in establishing probabilities for critical states of affairs (e.g., target classification and range),

- Alerting aids, which notify appropriate personnel when a preset critical threshold is exceeded by some indicator (e.g., the probability of being within counterdetection range),

- Prompting aids, which suggest and prioritize possible courses of action (e.g., approach maneuvers, weapon selection, time and method for communication, torpedo evasion maneuvers),

Figure 2-1 lists seven representative contexts in which a need for aids was identified and specifies for each the functions which an aid might perform.

2.3 Focus on Target Ranging

A particular decision context, target ranging, was selected for a more detailed conceptual specification of aids. This selection was motivated by the following criteria:

- the high stakes involved

- the frequency with which the problem arises (or is expected to arise in wartime)

- the perception by members of the fleet that an aid would be helpful

- the appropriateness of DSC's expertise to the development of the aid.

ILLUSTRATIVE AID NEEDS

DECISION/ASSESSMENT	AID FUNCTION
CLASSIFY TARGET	ASSESS PROBABILITIES
ESTIMATE TARGET RANGE	ASSESS DISTRIBUTION INDIVIDUAL* MULTIPLE* ALERT TO DANGER/OPPORTUNITY*
CLOSE THE TARGET	SUGGEST MANEUVERS/WEAPON SELECTION
FIRE TORPEDO	SUGGEST TIMING
EVADE TORPEDO(S)	SUGGEST CONTINGENT MANEUVER
RESPOND TO FLOODING	IDENTIFY REMAINING MBT BLOW OPTIONS
COMMUNICATE	SUGGEST TIMING AND METHOD

*CONCEPT DEVELOPED

Figure 2-1

3.0 DEVELOPMENT OF TECHNICAL CONCEPTS

3.1 Requirements of the Target Ranging Situation

Consider the following scenario: A U.S. nuclear-powered attack class submarine (SSN) is on a barrier patrol in unfriendly waters during wartime. Its mission is to detect and destroy transiting enemy submarines. Contact is established by passive sonar with a vessel which is classified as a hostile submarine.

The Officer of the Deck (often, though not always, the CO) needs a continuously updated best guess as to target range and an assessment of its probable accuracy. As noted in the introduction, critical decisions are based on these estimates. The CO will not order an attack in this offensive situation until he is reasonably sure that:

(a) the target is within range of the selected weapon,

(b) solution accuracy is good enough to bring the target within the search envelope of that weapon.

If these conditions are not satisfied, an attack will waste a valuable weapon and sacrifice the advantages of covertness. An alerted enemy may either take evasive measures or counter-attack (or both).

On the other hand, if the CO waits too long to launch an attack, he runs several risks as well. The opportunity for a kill will be lost if contact with the hostile submarine is lost, or if it moves out of his assigned zone. At the same time, the longer he waits, the higher the chance of counterdetection and a consequent loss of advantage.

3.2 Current Target Ranging Practice

How then is target range assessed? Solutions specifying target range (as well as course and speed) are computed by sonar, plot, and fire control. Each of these major divisions, moreover, has several techniques available within it. For example, sonar can employ Range of the Day, signal-to-noise ratio, and deflection/elevation angles. Plot encompasses geo plots, hyperbolic plots, time/range plots, and Ekelund. Fire control contains both KAST and MATE, as well as automated versions of Ekelund and D/E angles.

The CO, however, has no formal guidance in his handling of these various solutions. Confronted with a widely dispersed set of estimates (as in the Time/Range plot of Figure 3-1), he may be unable to settle on any single estimate at all, however tentative.

Typically, he selects the one solution he regards as most believable in the context and disregards the others. At best, he may informally select a solution intermediate between values he has confidence in. But to the extent that he does pool more than one solution, he has no formal way to assess the credibility of the pooled estimate as a function of his confidence in the original solutions.

To make matters worse, no systematic and general procedure is available for assessing confidence in a particular solution. Such a procedure would have to take account of numerous variables. These include quality of bearing data, geometry of own ship maneuvers, pattern of change in a solution over time, knowledge of the environment (bottom condition, sound velocity profile), and competence of operators. The credibility of each solution is affected in a different way by each of these factors.

BASIC TARGET RANGING PROBLEM

TIME/RANGE PLOT

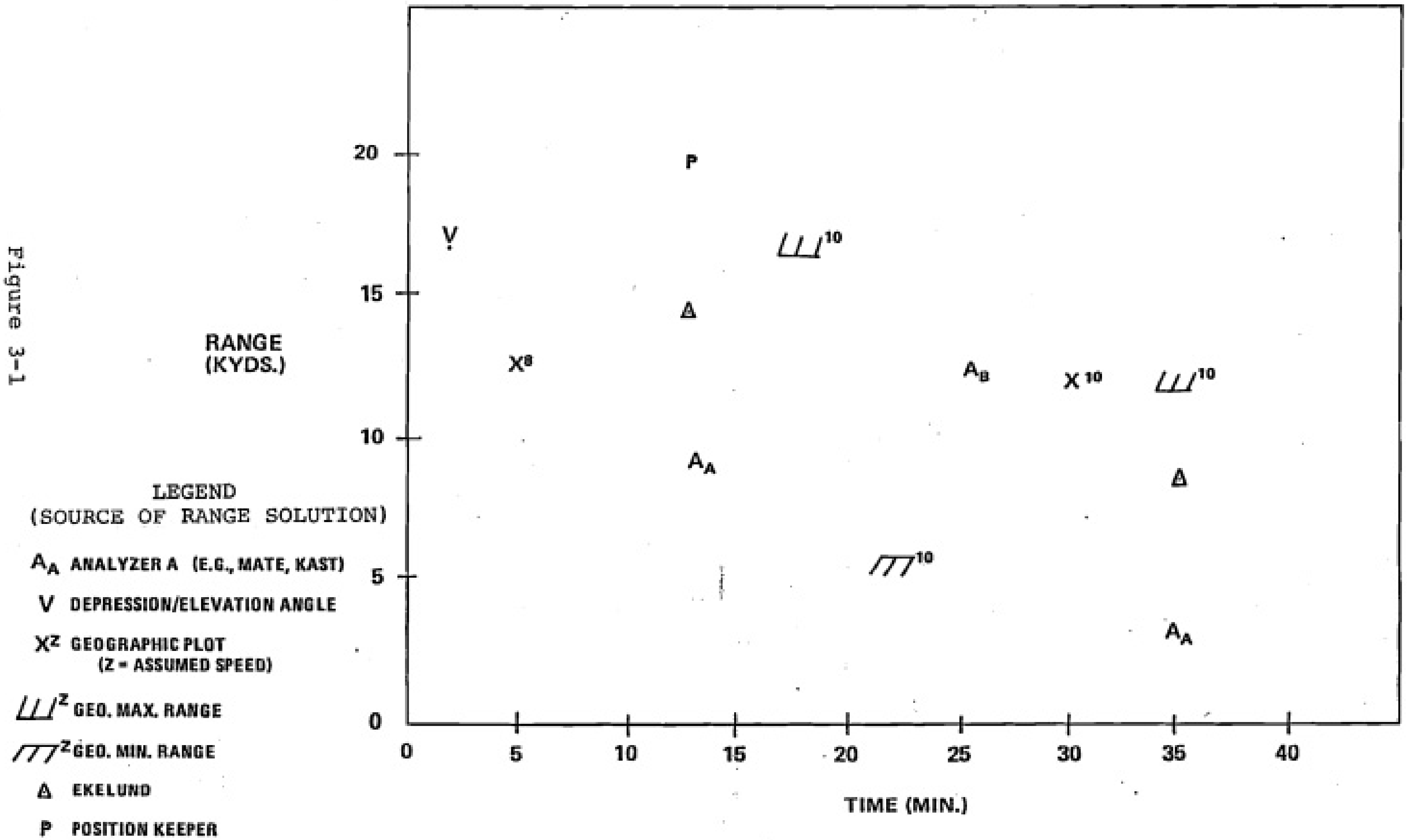


Figure 3-1

Decisions to launch an attack will be unnecessarily delayed if the increased precision of localization obtainable by pooling estimates is not utilized. Delay may also occur on account of cognitive overload--especially in multiple target scenarios with multiple solutions on each target.

Even so, decision-making is perhaps less affected by inaccuracy in the range estimate than by absence of an assessment of its accuracy. In principle, the Mark 48 torpedo can be fired at very long distances and with range errors as large as 20 to 50 percent. Such a weapon capability requires, for its full exploitation, a probabilistic rather than an absolute notion of target range. Crucial probabilities (e.g., of having an adequate solution and of being within weapon range) can be estimated from range error assessments without knowing very well where the target is. All too often, however, decisions to launch an attack are unnecessarily delayed while increased accuracy of range estimation is pursued.

We conclude that there is a prima facie need for decision aids which:

(a) assess confidence in particular solutions,

(b) produce a pooled estimate of target range together with an assessment of its precision,

(c) estimate critical probabilities (e.g., of being within weapon range) which form the basis for action.

DSC has developed concepts for three such aids.

3.3 Evaluating a Particular Solution: Decomposed Error Analysis

For each solution technique, this aid provides a probabilistic range assessment which takes explicit account of variable and fixed sources of error. Error in a particular range solution is decomposed into its contributing sources by a technique, Decomposed Error Analysis (DEA), developed by DSC staff (Brown, 1969). Figure 3-2 outlines the logic of the DEA decision aid, and Appendix B lays out its mathematical basis.

3.3.1 Output. The output of DEA is a probability distribution over possible target ranges based on evidence from a particular ranging technique. This may be more conveniently expressed as an expected range together with an interval within which the actual range should occur with a given probability (e.g., 95%). The size of that interval (or the spread of the distribution) is assumed to be inversely related to the degree of credibility of the expected range estimate produced by the relevant technique.

3.3.2 Input In general, each ranging technique encompasses an algorithm and certain primary readings to which the algorithm is applied. This algorithm and the primary readings are among the inputs to DEA (and are typically the only inputs required in current ranging practice). In addition, however, assessments of errors and dependencies among errors in primary readings are required as inputs to DEA. Error in the target range estimate is a function of these errors and correlations.

A residual error term is also assessed, which encompasses all remaining sources of error in the range estimate. Residual error corresponds to the error that would be expected

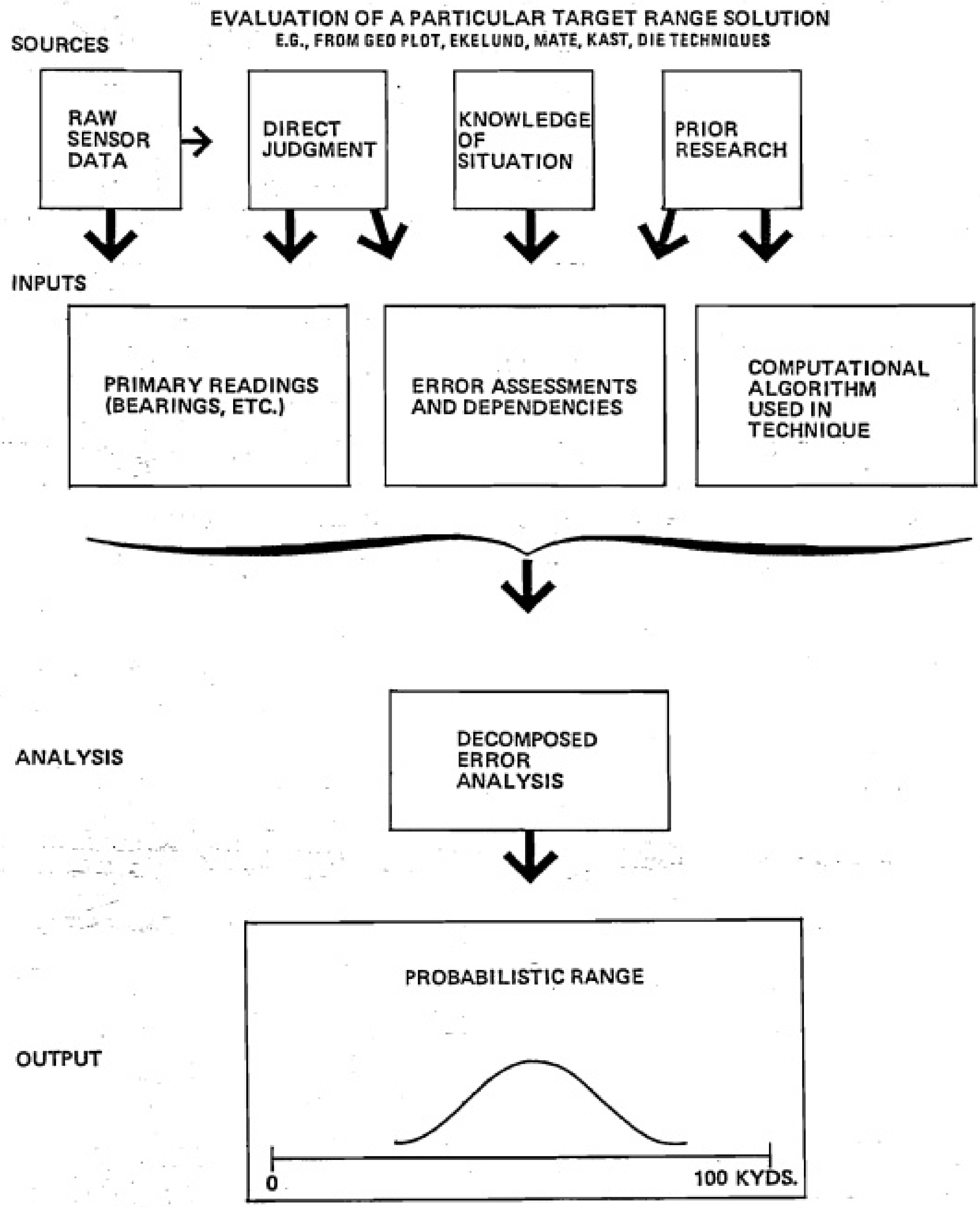

EVALUATION OF A PARTICULAR TARGET RANGE SOLUTION
E.G., FROM GEO PLOT, EKELUND, MATE, KAST, DIE TECHNIQUES
SOURCES
RAW SENSOR DATA
DIRECT JUDGMENT
KNOWLEDGE OF SITUATION
PRIOR RESEARCH
INPUTS
PRIMARY READINGS (BEARINGS, ETC.)
ERROR ASSESSMENTS AND DEPENDENCIES
COMPUTATIONAL ALGORITHM USED IN TECHNIQUE
ANALYSIS
DECOMPOSED ERROR ANALYSIS
OUTPUT
PROBABILISTIC RANGE
0
100 KYDS.

Figure 3-2

even if all primary readings were accurate. It thus captures the extent to which the assumptions of a technique fail to correspond to the situation in which it is applied, as well as the likelihood of computational mistakes or operator biases.

Assessments of error and dependency will not be constant across all the conditions in which target ranging takes place. Properties of the signal (e.g., relative bearing, bearing rate, signal-to-noise ratio), of the environment (e.g., sound velocity profile, ocean depth) as well as the number and type of maneuvers, may affect the size and direction of both. It is, therefore, necessary to supply values for an array of potential conditions.

3.3.3 Sources. What are the sources of the inputs required for DEA? The objective, of course, is to reduce, not increase, the burden of the CO and his staff. On the other hand, an entirely automatic procedure, in which no interaction at all is allowed for, is less likely to be trusted or to be used appropriately. Moreover, the CO and his staff may bring insights to a situation which are not captured in prior research. An accommodation of both considerations can be achieved by automatically providing default values for all inputs, while allowing those values to be overridden and replaced at the option of the command staff.

In each case, the personnel who make these adjustments should be the ones with the fullest information about the relevant variable.

For example, primary readings are automatically registered within the Fire Control System from the relevant sensors. But provision is made for an editing function exercised by an operator who may eliminate "bad" data points. In the case of manual techniques, of course, direct judgment always mediates the recording of data from sensors.

Default values for errors and dependencies can be largely based on prior research. Appendix E describes how Rangex AUTEC data may be used to compare "actual" values (e.g., of bearing rate) with values estimated on-board ship in order to derive the error and dependency estimates required. Separate estimates may be obtained for a variety of conditions (e.g., thermal). Knowledge of the current situation would be used on-board ship to retrieve the values which are appropriate at a given time. Most of the relevant properties of the data or the environment can (like the primary readings themselves), be automatically registered by shipboard sensors. In turn, the appropriate error and dependency values can be automatically retrieved.

Nonetheless, the CO or other members of his staff might wish to adjust an assessment of error or dependency on the spot, if aspects of the current situation are unique or if any other considerations cause him to disagree with the conclusions of prior research.

Thus, the proposed DEA aid systematically integrates objective and subjective information. It allows the CO to set a balance--governed by the prevailing time constraints and his own individual preferences--between guidance by prior research and dependence on his own intuitions. At the same time, it synthesizes the different types of expertise on board ship--bringing each to bear where it is most appropriate.

3.3.4 Worked example. A worked example of the application of DEA to Ekelund ranging is given in Figure 3-3. All data are hypothetical, but are intended to fall well within the range of probability.

3.3.4.1 Current Approach: Inputs and Outputs. According to the Ekelund formula target range (R_T) in yards is estimated by:

$$R_T = \left(\frac{Sx_1 - Sx_2}{\dot{B}_2 - \dot{B}_2}\right) \quad 1934$$

WORKED EXAMPLE FOR EVALUATION OF EKELUND SOLUTION

CURRENT APPROACH

INPUTS

COMPUTATIONAL ALGORITHM

$$R_T = \frac{Sx_1 - Sx_2}{\dot{B}_1 - \dot{B}_2} \cdot 1934$$

PRIMARY READINGS

Sx_1	Sx_2	$\dot{B}_1$	$\dot{B}_2$
15	-14	2	-2

OUTPUT

$\hat{R}_T = 14{,}022$

EVALUATION AID

PRIOR RESEARCH + DIRECT JUDGMENT

ADDITIONAL INPUTS

ERROR ASSESSMENTS

Sx_1	Sx_2	$\dot{B}_1$	$\dot{B}_2$	RESIDUAL
$+1 \pm .8$	$-1 \pm .8$	$0 \pm .5$	$0 \pm .5$	$0 \pm 1{,}500$

ERROR DEPENDENCIES

Sx_1, Sx_2	$\dot{B}_1, \dot{B}_2$	$Sx_1 - Sx_2, \dot{B}_1 - \dot{B}_2$
-.33	+.5	-.15

OUTPUT

$\hat{R}_T = 15{,}051 \pm 2{,}508$

Figure 3-3

where Sx_1 and Sx_2 are own ship speed across the line of sight (knots) in the first and second legs of a maneuver, respectively; and $\dot{B}_1$ and $\dot{B}_2$ are bearing rates (degrees/second) for the two legs. These quantities constitute the "primary readings".

Current practice consists in the application of this algorithm, either manually or within the Fire Control System, to the primary readings, on the assumption that no target maneuver has been detected. In the example given, the range estimate produced by that means would be 14,022 yards. No indication of confidence in the solution is provided.

3.3.4.2 Evaluation Aid: Inputs. Error assessments for the primary readings incorporate two terms: Bias is the expected error, i.e., the expected difference between the true values and readings on board ship. Secondly, the interval of uncertainty reflects the variability of errors in ship-board readings. In the example of Figure 3-3, sensors (plus some auxiliary calculations) produce a reading for speed across line of sight on the first leg of 15 knots. In order to compensate for bias, a correction term of +1 knot, based on prior research or direct judjment, is added to this figure. The expected value of Sx_1 is thus 16 knots. And the true value of Sx_1 falls with 95% certainty within the interval 16 $\pm$ .8 knots.

Residual error, as noted, may be due to violation of the assumptions necessary for perfect accuracy of the computational algorithm. Ekelund ranging, for example, requires in principle a motionless target. (In practice of course, it often provides a tolerable approximation to the true range.) Residual error, too, consists of a bias term and an interval of uncertainty, conditioned on prevailing circumstances.

Dependencies are also assessed: between bearing rate error on one leg and bearing rate error on the other; between speed across line of sight on one leg and the other; between the change in bearing rate from one leg to the next and the change in speed across line of sight. Dependency may be expressed either as a regression coefficient, as in Figure 2-4, or as a correlation.

3.3.4.3 Evaluation Aid: Output. The output of the DEA aid is an adjusted estimate of target range together with an interval of uncertainty.

Note that the adjusted target range (15,051 yards) is over a thousand yards greater than the figure that would have been arrived at without the aid. There are two factors underlying the adjustment. First, and most obviously, the aid corrects for bias in the readings of speed across line of sight. A second, more subtle cause of the upward adjustment is the variability in bearing rate estimates. According to the Ekelund formula, target range is a non-linear function of change in bearing rate. In general, the expected value of a quotient is not the quotient of the expected values, when there is significant error of measurement in the denominator. (See formula [4] in Appendix B.) A third potential cause of adjustment--residual bias--does not occur in this particular example.

The interval of uncertainty tells us that, if we had only Ekelund ranging to rely on in assessing target range, we could be 95% sure that target range falls between 12,543 and 17,859 yards.

3.3.4.4 Degree of Decomposition. It should be noted that the level to which error decomposition is carried (i.e., the "primary readings") is somewhat arbitrary. Thus, bearing rate error can be further decomposed into errors in bearing readings. And error in speed across line of sight can be expressed in terms of error in measures of own ship course and speed, as well as bearings. The chosen decomposition should be one for which convenient sources of input are available from prior research and for which subjective adjustments tend to be natural and accurate.

Subjective adjustments, however, are not confined to a single level of decomposition. The appropriate personnel might use direct judgment to adjust the interval of uncertainty (or bias) for bearings, for bearing rate, for change in bearing rate, or even for the final output itself, target range.

3.4 Pooling Different Solutions

The results of the separate passive ranging techniques are pooled by this aid to produce a single probabilistic range assessment. The method takes account both of the (shifting) relative validity of the different techniques and the degree of overlap or redundancy in their sources of information (Brown and Lindley, 1978; Lindley, Tversky, and Brown, 1979; Freeling 1980). The output reflects, in a readily understood way, all the available sources of information on target range. Figure 3.4 outlines the logic of this aid, and Appendix C sketches its mathematical basis.

3.4.1 Output. The output of the reconciliation aid is a probability distribution over possible target ranges,

POOLING DIFFERENT TARGET RANGE SOLUTIONS

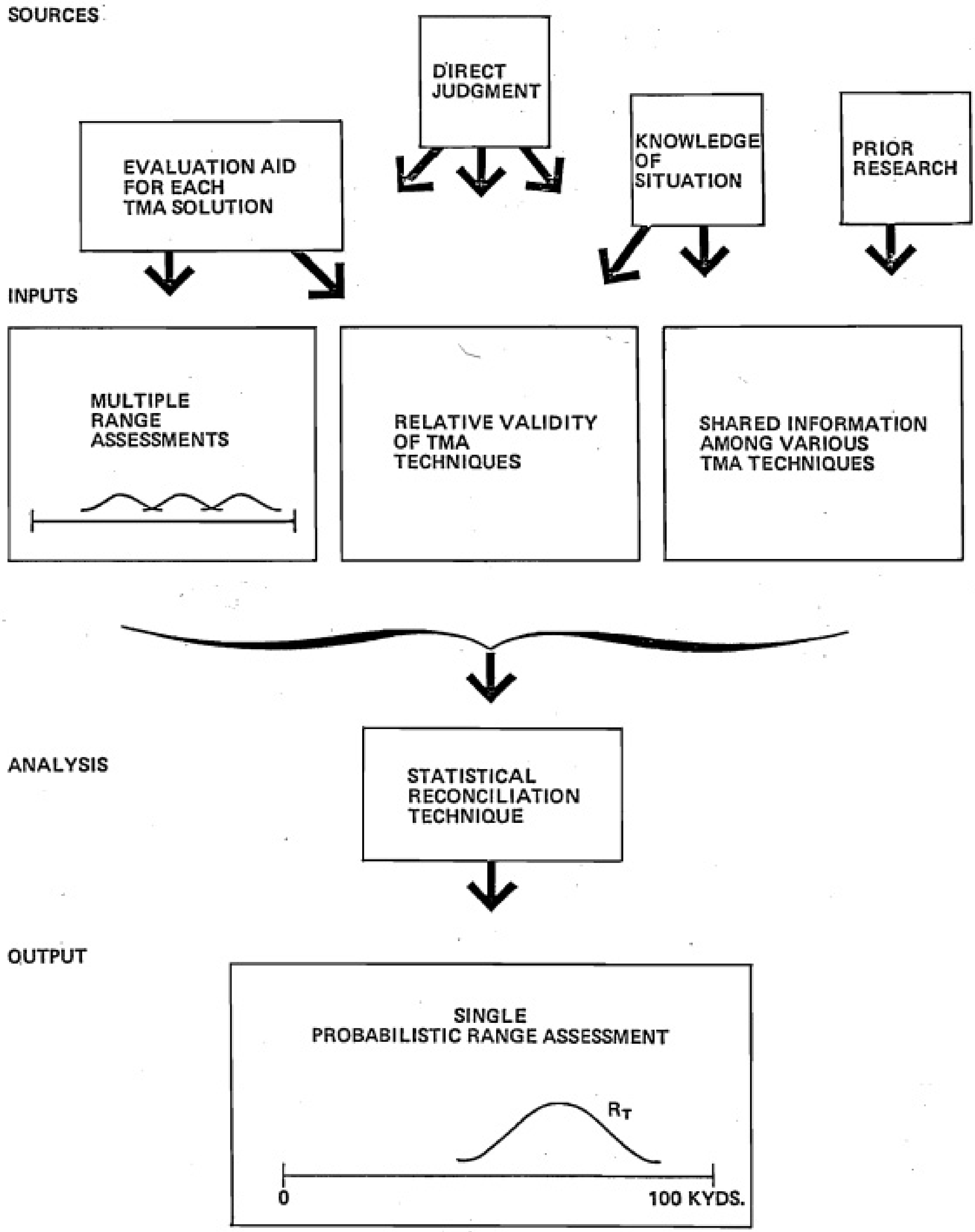


Figure 3-4

based on evidence from all available ranging techniques. It may be summarized by an expected range and a credible interval, within which the true target range occurs with a given degree of certainty (e.g., 95%).

3.4.2 Inputs. The primary input consists of the range estimates from the various techniques. Also required are assessments of their relative credibility and of their interdependencies. As noted above, credibility can be represented as a function of the interval of uncertainty characterizing the range estimate from a given technique. Interdependency, given certain assumptions (Appendix C), represents the degree of correlation between the errors in two techniques.

Both credibility and interdependency must be considered by an adequate reconciliation procedure. The more credible a technique is, the more weight it receives in determining the reconciled estimate, and the more it contributes to the quality of the reconciled estimate. On the other hand, if a technique draws on data which are already exploited by other techniques, its impact on the solution is reduced and there is less enhancement of the credibility of the output.

Reconciliation is thus not a process of determining which range technique is likely to be best on a given occasion. A technique which tends to be less accurate may, nevertheless, have something to contribute. Intuitively, the reason is that it draws on sources of information or evidence which other techniques do not tap. The proposed method captures this intuition by assigning each solution a weight based in an approximate sense (Appendix C) on the information accessed exclusively by that technique. Information common to two techniques tips the scales in favor of neither one nor the other (Freeling, 1980).

Two aspects of information can be logically distinguished: the sensory data (the "primary readings"), and information about the relation between sensory data and the variable of interest, target range. These two aspects of information correspond to the broad decomposition of sources of error by the evaluation aid into errors in primary readings, on the one hand, and residual error, on the other. The same classification applies to the interdependence between two techniques. Errors may be statistically related when common assumptions (e.g., no target maneuver) or common variables (environment, signal, nature of maneuver, operator bias, etc.) condition the accuracy of the two sets of primary readings, on the one hand, or the two sets of algorithms, on the other.

3.4.3 Sources. Inputs for the reconciliation aid are derived from a mixture of prior research, sensing of prevailing conditions, and direct judgment.

Decomposed error analysis can, of course, provide many of the required inputs. The evaluation of each ranging technique yields an adjusted range estimate and a measure of validity for the solution from that technique. The reconciliation aid, however, need not be coupled with DEA. Each ranging technique, as currently practiced, provides its own estimate of target range. Adjustments for bias and estimates of relative validity can be directly assessed either by operators or by command personnel (or both).

Interdependencies among ranging techniques can be estimated from prior research (Appendix E) subject to override by relevant personnel. Like the other inputs discussed here, the degree and direction of interdependency may depend on properties of the signal, the environment, or the nature of maneuvers. Thus, default values corresponding to different conditions could be stored, and the values appropriate to each situation retrieved.

3.4.4 Worked Example. Figure 3.5 presents a worked example for the range pooling aid. Data are hypothetical but presumed plausible.

3.4.4.1 Current Approach. Consider a somewhat more detailed version of the previously described scenario. A U.S. attack submarine is on barrier patrol in unfriendly waters. Its mission is to engage hostile submarines. A contact is classified as a Soviet diesel submarine on the snorkel (i.e., using diesels to recharge its batteries). The Range of the Day (ROD) is 15,200 yards (i.e., the expected range at first contact for this type of target under current conditions). The sonarman concludes on the basis of sound intensity propagation loss that the contact is significantly closer, probably having entered well within detection range while operating quietly on the battery. Taking both propagation loss and ROD into account, the sonarman assesses target range as 8,000 yards.

In the meantime an Ekelund range has been computed as 14,022 yards; and a range estimate based on Deflection/Elevation angle is 9,650 yards.

The officer of the deck currently has no formal guidance in arriving at a single range estimate from these discrepant estimates.

3.4.4.2 Reconciliation Aid: Inputs. Error assessments for Ekelund are derived, as previously described, from the DEA evaluation aid. D/E bias and credible interval represent another relatively straightforward application of DEA. It is not as easy to decompose the sonarman's judgment, which is based on apparent sound intensity and a tentative classification (as well as ROD). Nonetheless, bias and credible

WORKED EXAMPLE FOR POOLING OF RANGE SOLUTIONS

CURRENT APPROACH

INPUTS

MULTIPLE RANGE ASSESSMENTS

ROD	15,200
SONARMAN	8,000
EKELUND	14,022
D/E	9,650

OUTPUT $\hat{R}_T$ = ?

RECONCILIATION AID

PRIOR RESEARCH + DIRECT JUDGMENT

ADDITIONAL INPUTS

	BIAS ADJUSTMENT	95% CREDIBLE INTERVAL
SONARMAN	0	+6,000, -4,000
EKELUND	+1,029	±2,508
D/E	-450	±3,500

ERROR DEPENDENCIES

A	B	
SONARMAN	ROD	1.0
ROD, SONARMAN	EKELUND	0
ROD, SONARMAN, EKELUND	D/E	.5

OUTPUT $\hat{R}_T$ = 12,724 +2,330, -1,916

Figure 3-5

intervals might be estimated directly from prior research--subject, of course, to adjustment on the spot by the sonarman himself or by the command staff (or both). As a result of these inputs, we have putatively unbiased range estimates from each solution source together with an interval of uncertainty.

In order to estimate interdependencies, regression coefficients of errors are assessed between pairs of techniques. This is an iterative procedure in which one member of the pair might be the result of reconciliation at the previous stage. Figure 3-5 shows the slope for errors in technique B regressed on errors in technique(s) A.

Interdependencies of errors between techniques can be estimated from prior research in the form of correlations (Appendix E). Subjective assessment or adjustment of correlations is, however, difficult to perform in a consistent way. Some very preliminary research suggests that, under certain conditions, a reasonable analog to the regression coefficient may be provided by the notion of shared information (Appendix C). The slope of errors in technique B versus errors in technique A can be roughly described as "the proportion of information in B which is also in A." Interdependencies may be assessed or adjusted more naturally in terms of shared information, and then converted to correlations for use in the reconciliation algorithm. Thus, referring to Figure 3-5, since all the information in the Range of the Day was incorporated into the sonarman's judgment, the assessment is 1.0. Ekelund ranging and sonarman's judgment are judged (illustratively) to share no information, while 50% of the evidence for D/E range is subsumed in the combined evidence for the sonarman's judgment and for the Ekelund range.

3.4.4.3 Reconciliation Aid: Output. The reconciled probalilistic estimate of target range is 12,724, + 2330 or -1916, yards.

Note that the interval of uncertainty for the pooled estimate is less than that for any of the contributing techniques.

This is a direct result of the fact that the techniques it draws on are not wholly redundant. Thus, the reconciled estimate is based on a larger fund of data than any particular range solution. Systematic integration of multiple solutions can lead to a more precise localization of the target--hence, perhaps, to an earlier time of fire.

On the other hand, the proposed method guards against an unwarranted sense of certainty. Evidence that is shared is not counted twice. When solutions do converge, it can be a dangerous error to suppose that one solution independently confirms another if they in fact rest upon the same data.

Even in current practice, some integration of range solutions takes place. For example, MATE is an interactive program which allows an operator to evaluate proposed range solutions. If he is aware of solutions from other techniques, they may influence the hypotheses he tests. The currently proposed aid is not incompatible with this procedure. On the contrary, it provides a systematic framework for assessing its true impact. The informational value of a range assessment technique will depend on the degree to which it draws on information not already utilized in other techniques.

Another example, the time/range plot (Figure 3-1), is particularly important, since it is often relied on by a CO to informally reconcile range estimates. Current range can be assessed by fitting a line by eye to range solutions plotted against time and extrapolating to the present. This method of reconciliation, however, suffers from several drawbacks:

- it does not formally provide for weighting the different solutions by a measure of their credibility.
- it does not allow for redundancy. Convergence of solutions is not a good measure of confidence, since it may be due to correlation of errors rather than increased accuracy.
- it fails to exploit the information about target course and speed provided by various TMA techniques.

An alternative approach is to update past range solutions, using estimates of course and speed, before pooling them. Such a procedure is sketched in Appendix B ("Updating").

3.4.5 <u>Sample display</u>. Figure 3-6 incorporates the worked example for the range pooling aid and suggests one form in which its graphic output might be displayed. Probabilistic assessments of target range from particular techniques are presented at the top of the display. The reconciled probabilistic estimate of target range is presented at the bottom.

Figure 3-7 depicts a subsequent phase of the scenario. At 13:20 a new estimate from the sonarman is available, as well as a new D/E angle. In addition, we now have estimates from geo plot and KAST. In this scenario, sonar, plot, and fire control agree the target is closing, and target speed is estimated from turn count as 4 knots. The original Ekelund estimate for 13:05 (shown by dotted line) has been updated by reference to these estimates of target course and speed.

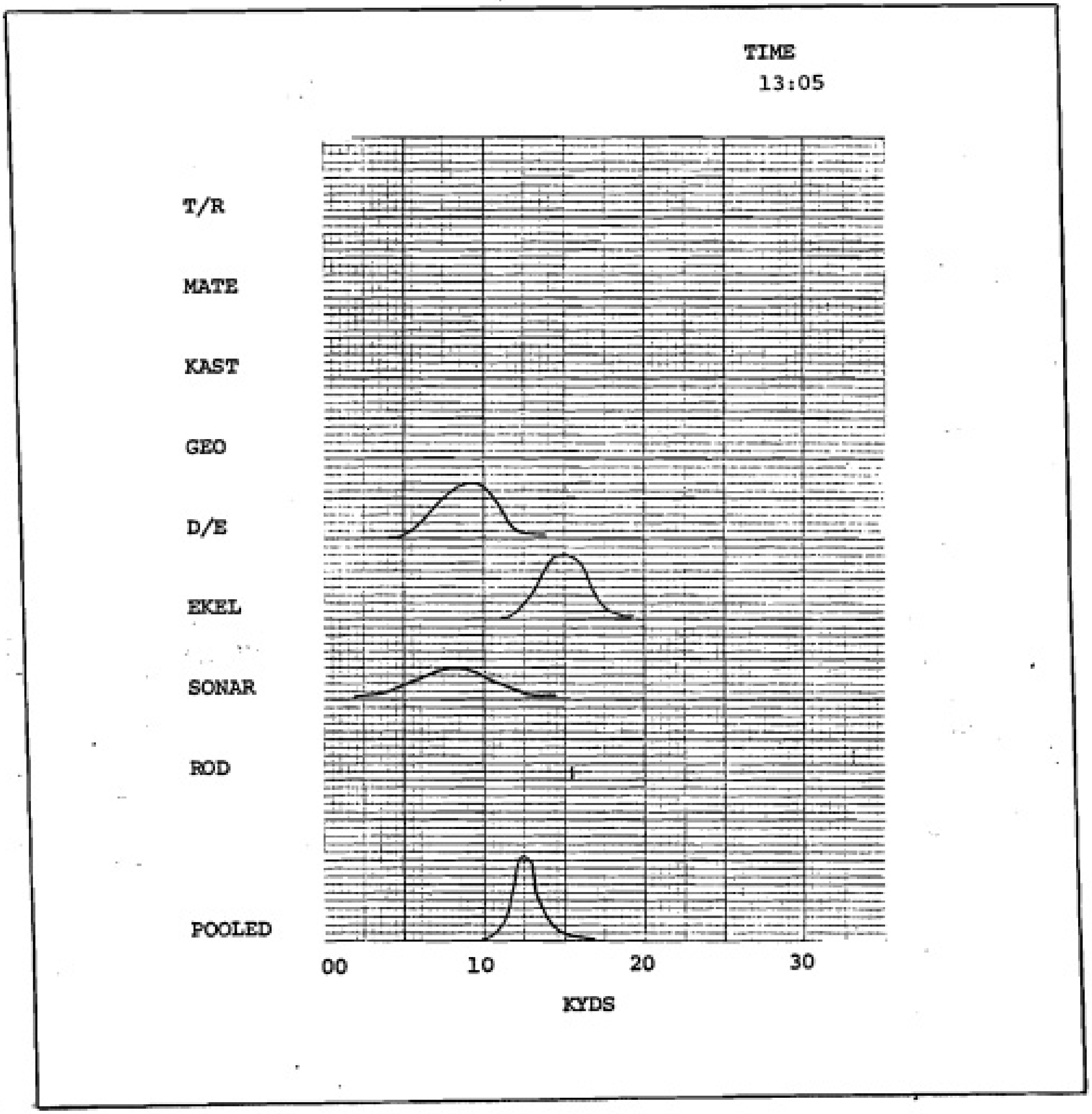
3.4.6 Updating. As this example suggests, the pooling aid does not require that all range estimates be originally computed for the target as it was at a single point in time. Such an assumption would not be particularly restrictive for "instantaneous" techniques like D/E and propagation loss. In these cases, solutions for current range based on fresh data either are or can be available at any time. It is restrictive, however, for a technique like Ekelund which requires a specified set of own ship maneuvers and assesses range for a particular time during those maneuvers. Similarly, cumulative techniques like KAST and MATE, while in principle always up to date, can be quite untrustworthy when new data are not coming in.

The method of updating proposed here should be distinguished from mere dead-reckoning on the basis of current course and speed estimates. Rather, it uses DEA to take account of uncertainty in the speed and course estimates which are employed. Thus, in our example, the credibility of the Ekelund range is reduced after updating. Appendix B gives the mathematical basis for this application.

3.5 Alerting at Critical Ranges

The probabilistic target range assessment is used by this aid to alert the CO to critical dangers or opportunities: e.g., when the probability that the target is within weapon range exceeds a preset threshold.

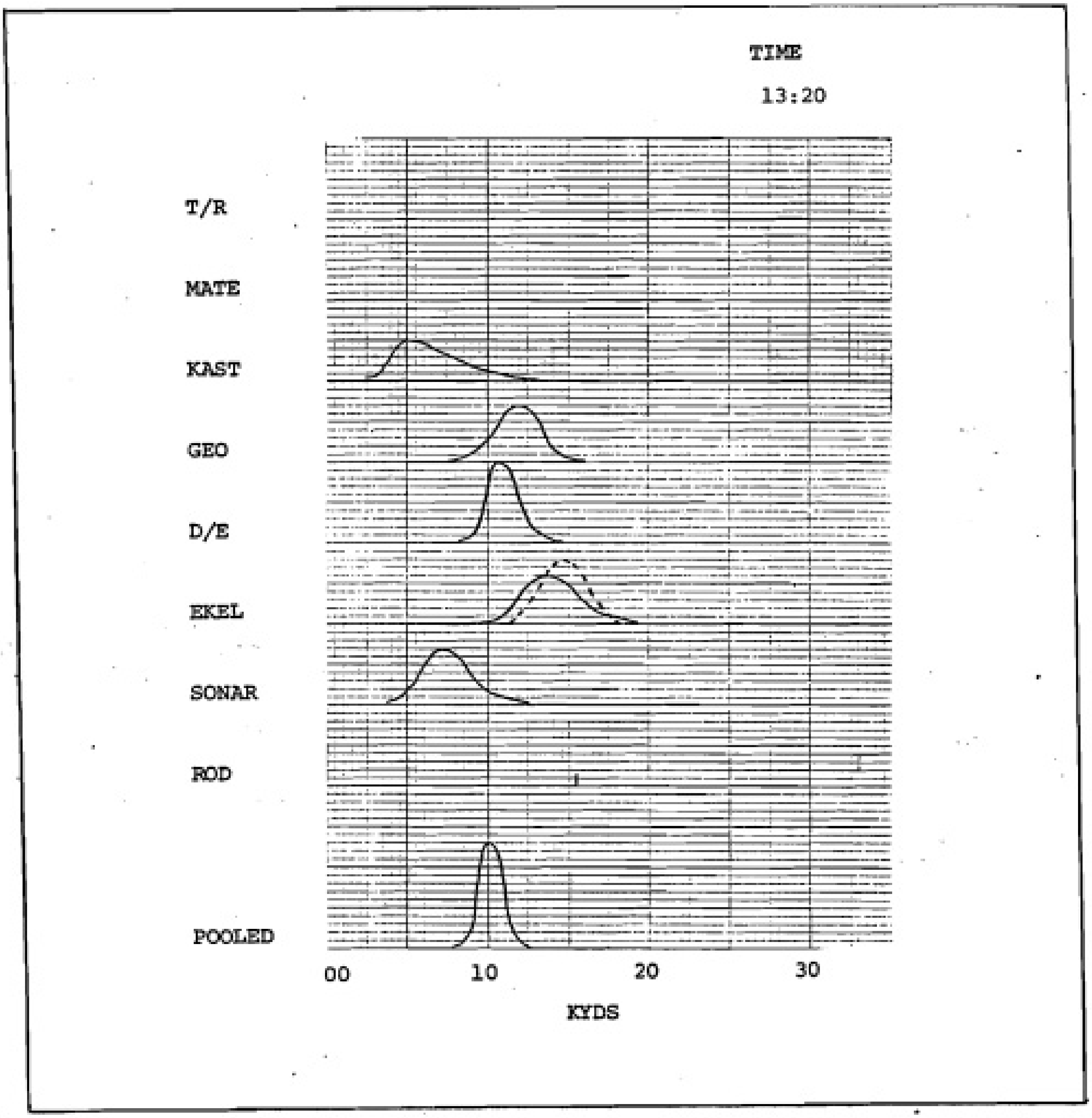
Alerts might be based on other critical probabilities as well: e.g., the probability that own ship is within target weapon range and the probability that own ship is within counterdetection range. Figure 3-8 shows the mechanism of such an aid, and Appendix D gives its mathematical basis.



RANGE POOLING

SAMPLE DISPLAY 1

Figure 3-6



RANGE POOLING

SAMPLE DISPLAY 2

Figure 3-7

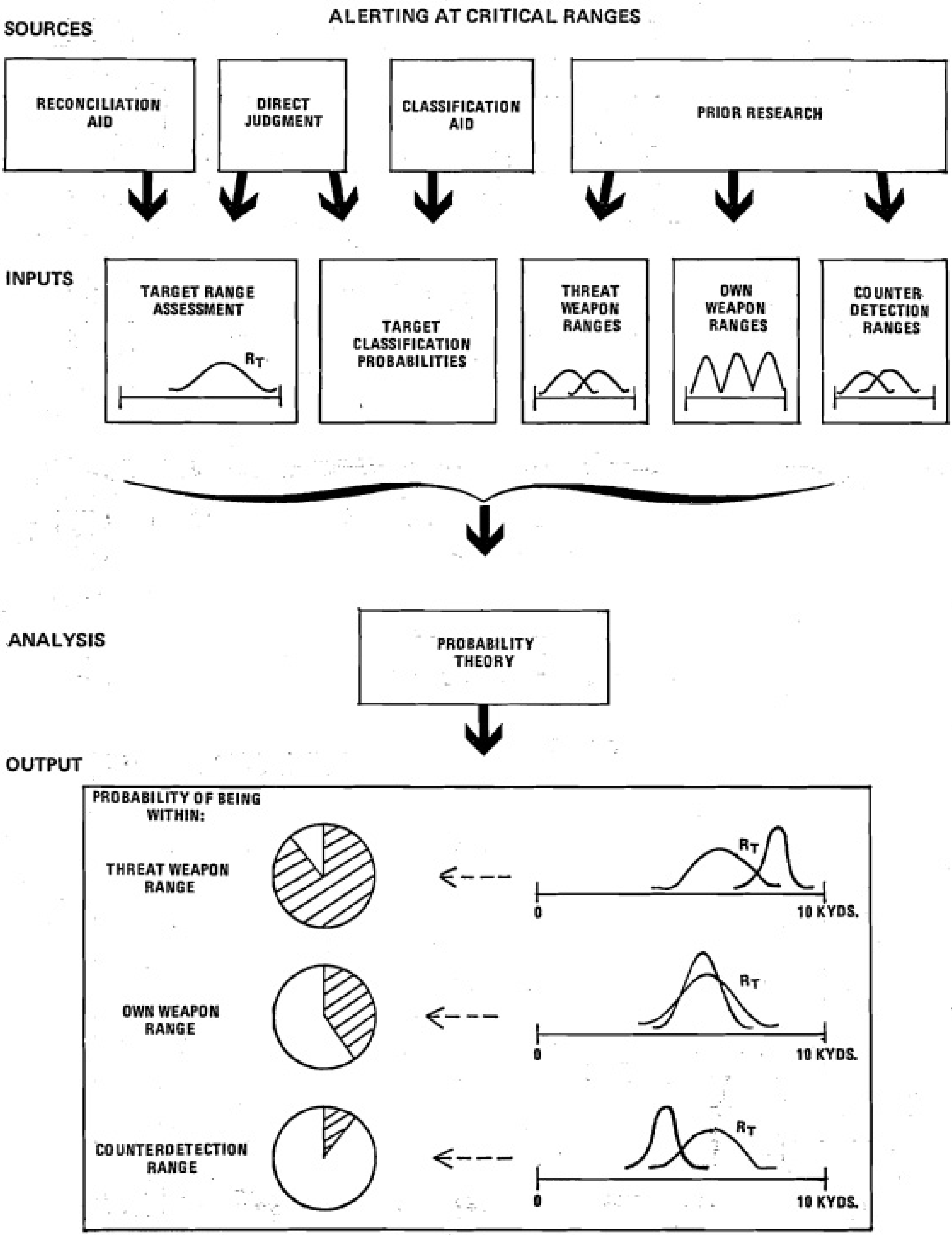
3.5.1 Output, input. The output of this aid is a display of the critical probability together with an alerting signal when that probability exceeds the criterion. In addition, there is a display of the distributions from which the critical probability was computed. These source distributions include the probabilistic target range estimate (R_T) and an assessment of target weapon range, target counterdetection range, or own ship weapon range, as the case may be.

The assessments of target capabilities depend, of course, on a classification of the target. The output of the aid might be broken down according to the possible target classifications. Or, at the option of the user, it might provide a single distribution by probabilistically combining the assessments based on all target classifications.

3.5.2 Sources. Target range assessments might be derived from an aid like the one previously proposed. But they can also originate from any of the ranging techniques as currently practiced. Similarly, classification probabilities might be based on a systematic inference aid, or else on direct judgment and currently available intelligence. Assessments of enemy and own ship capabilities will be derived from prior research.

3.5.3 Worked Example. Figure 3-9 presents a worked example of the alerting aid, using hypothetical data.

3.5.3.1 Inputs. The scenario introduced previously is reviewed and extended. Contact has been established with a Soviet diesel sub, whose range and 95% credible interval are estimated at time 13:05 as 12,724 yards (+2330, -1916). We assume the CO has tentatively decided to fire when the



ALERTING AT CRITICAL RANGES
SOURCES
RECONCILIATION AID
DIRECT JUDGMENT
CLASSIFICATION AID
PRIOR RESEARCH
INPUTS
TARGET RANGE ASSESSMENT
R_T
TARGET CLASSIFICATION PROBABILITIES
THREAT WEAPON RANGES
OWN WEAPON RANGES
COUNTER DETECTION RANGES
ANALYSIS
PROBABILITY THEORY
OUTPUT
PROBABILITY OF BEING WITHIN:
THREAT WEAPON RANGE
R_T
0
10 KYDS.
OWN WEAPON RANGE
R_T
0
10 KYDS.
COUNTERDETECTION RANGE
R_T
0
10 KYDS.

Figure 3-8

WORKED EXAMPLE FOR ALERTING IF WITHIN O/S WEAPON RANGE

ALERTING AID

INPUTS

TARGET RANGE ASSESSMENTS

Time	Range	Credible Interval
13:05	12724	+2330, -1916
13:30	6500	+556, -490
13:41	5600	+405, -350

O/S WEAPON RANGE 6000

C.O.'S THRESHOLD FOR FIRING: PR [TARGET RANGE LESS THAN O/S WEAPON RANGE] ≥ .90

OUTPUT

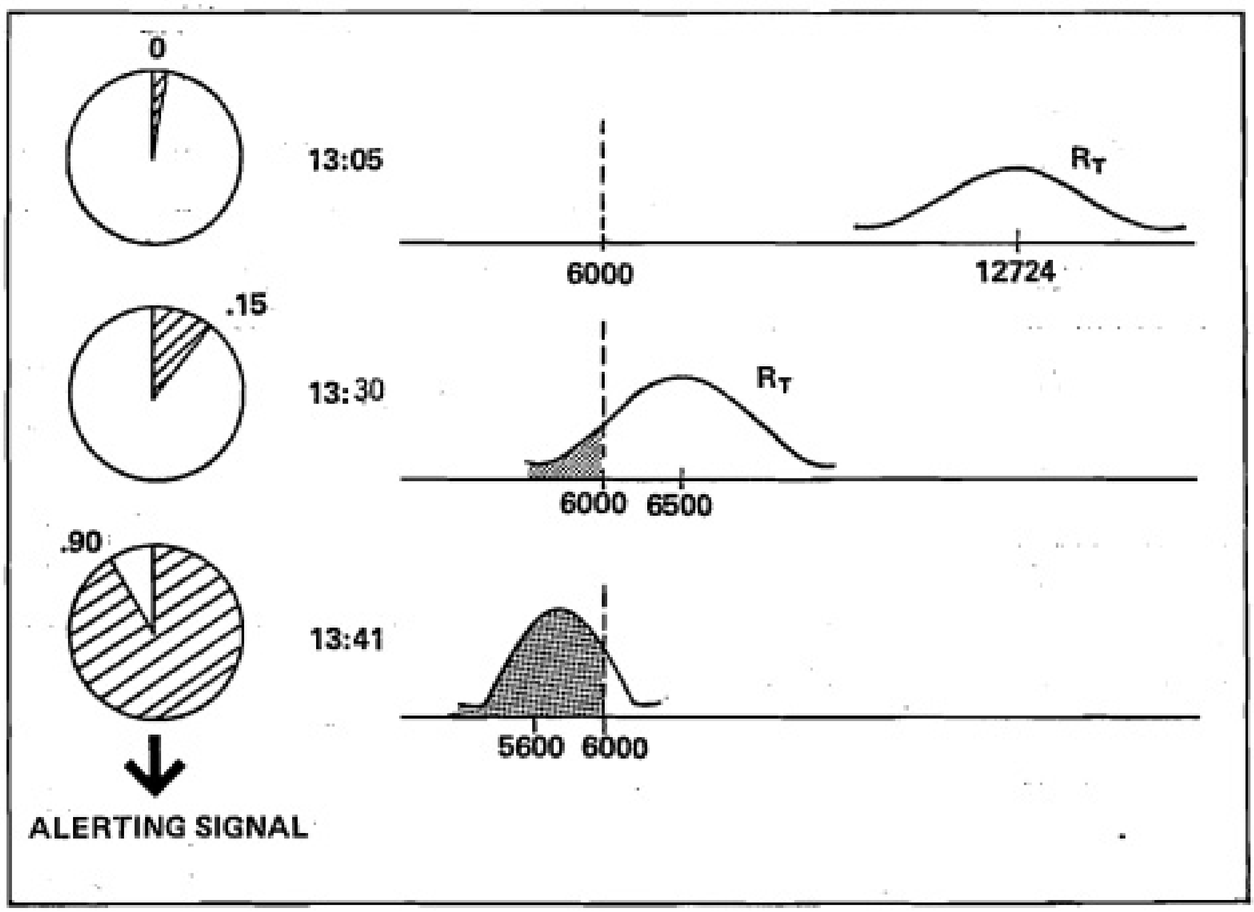


Figure 3-9

probability that the target is within 6,000 yards exceeds 90%. At time 13:30, target range is assessed as 6,500 yards (+556, -490). The target continues to close until at 13:41, range is estimated to be 5,600 yards (+405, -350).

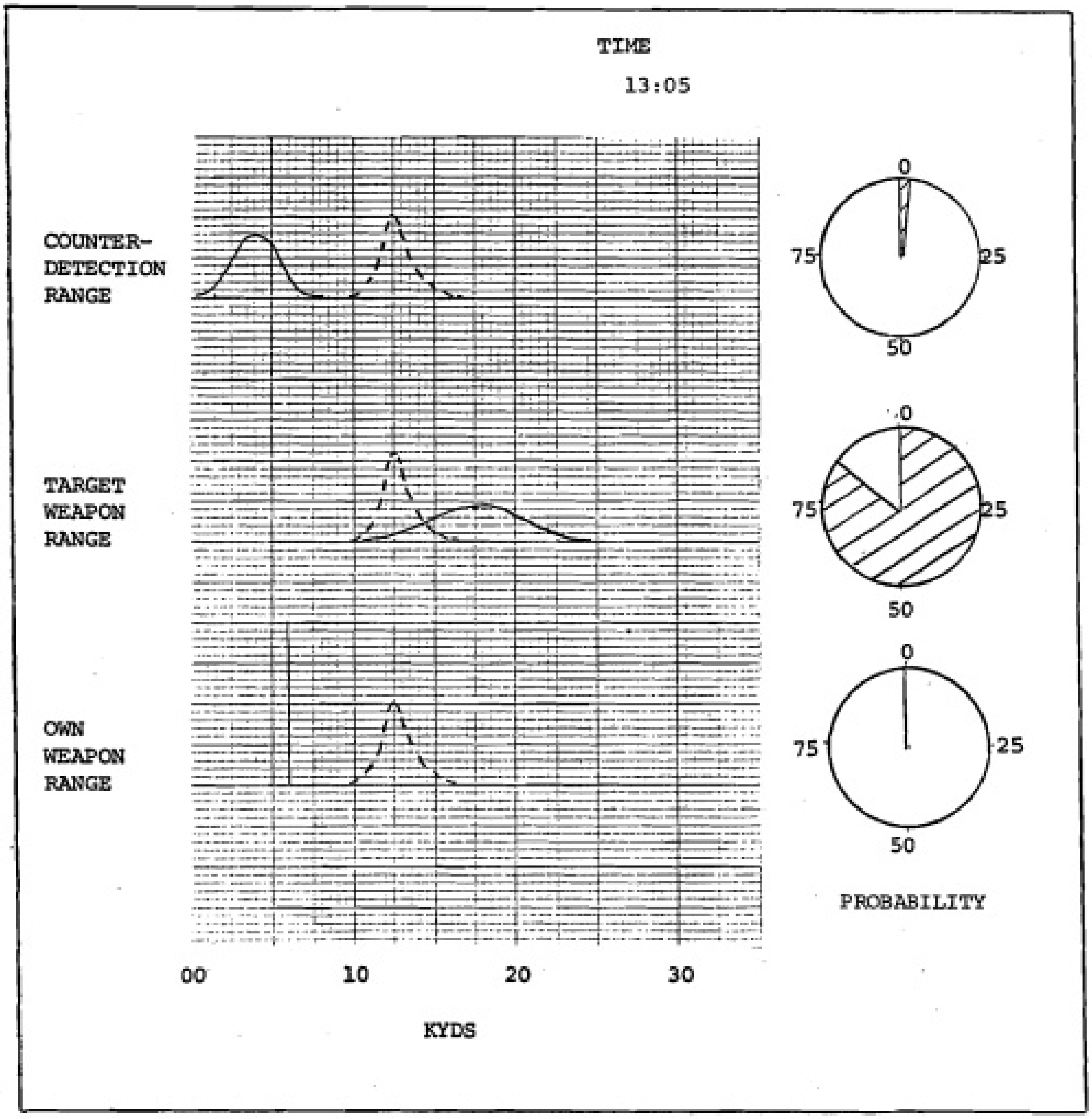
3.5.3.2 Output. The dials show how the probability of being within weapon range (6,000 yards) changes with time. At 13:05, this probability is negligible; it is about 15% at 13:30; finally at 13:41 the criterion of 90% is reached, and an alert is sounded.

At the same time, the shift in the location of the target range distribution which underlies these changes can also be viewed. Note as well how this distribution becomes tighter as solution quality improves.

Suppose the CO requires, as a further condition for firing a torpedo, that solution accuracy be within 1000 yards with 95% certainty. An additional signal might inform him that this condition, too, is fulfilled at 13:41.

3.5.4 Sample displays. Figures 3-10 and 3-11 show more concretely how critical probabilities and source distributions might be displayed.

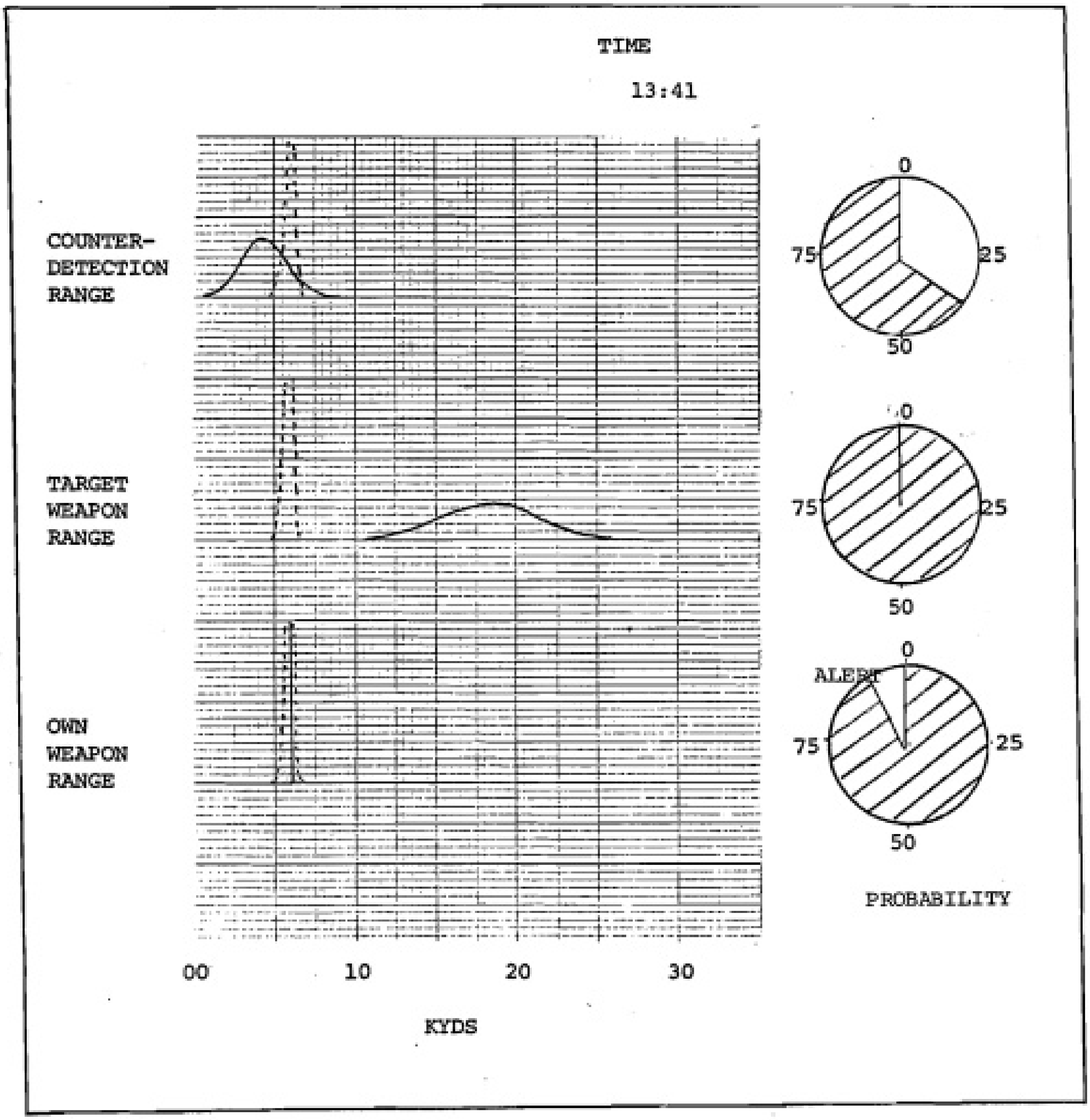
The distribution of target range is depicted in dotted lines. The solid distributions represent target counterdetection range, target weapon range, and own ship weapon range. Note that target range decreases and is measured more precisely at 13:41 (Figure 3-11) than at 13:05 (Figure 3-10), while the solid distributions remain fixed. In this illustrative situation, enemy weapon range is regarded as greater than enemy counterdetection range.



ALERTING

SAMPLE DISPLAY 1

Figure 3-10



ALERTING

SAMPLE DISPLAY 2

Figure 3-11

4.0 CONCLUSION

4.1 What Has Been Done

The proposed aids appear to satisfy, on a conceptual level, the requirements that motivated them. They provide:

(1) a range estimate which is based on all the available data

(2) an assessment of its credibility which is explicitly derived from component sources of error and interdependencies

(3) the implications of the assessments for critical probabilities which are relevant to action.

Expected benefits include the following:

(1) Improved accuracy of range estimates, for a given amount of raw data

(2) Command staff no longer obliged to get involved in analysis in order to assess quality of a solution

(3) More timely decisions based on quality of solution and on critical probabilities.

These aids represent a synthesis of objective and subjective inputs. On the one hand, their input is subject to continuous automatic updating. On the other hand, the values of parameters can be interactively adjusted by command personnel when unique circumstances or other considerations cause them to disagree with the automatically provided values. The aid is not, therefore, another "black box". The basis for confidence in its output should, with proper training, be quite clear to those who use it.

4.2 What Remains To Be Done

Demonstration of the value of implementing such aids depends, of course, on several further steps: i.e., quantification, testing, integration within the combat system, and refinement.

(a) Preliminary quantification. An examination of data from Rangex TMA exercises suggests that the quantitative assessment of parameters necessary for the aids is feasible (Appendix E). Such parameters include statistics (expected values, variances, and covariances) on errors in bearing rate, and own ship speed, as well as on the error introduced by violations of assumptions in the various TMA techniques.

Further study of the data from Rangex and other sources may improve the precision with which those parameters are assessed. Particular attention must be paid to the task of identifying possible conditioning variables, whose values affect the values of aid parameters. Such conditioning variables might include, for example, signal-to-noise ratio, features of the sound velocity profile, and whether or not there is a maneuver by own ship across the line of sight.

(b) Validity. Once aid parameters have been assessed, the performance of the inference aids can be evaluated. Assessments of target range produced by the aids can be compared with the more exactly reconstructed ranges from land-based sensors recorded in Rangex AUTEC data. Current practice on board the submarines can also be compared with the reconstructed ranges. Only if the proposed range estimation technique approximates true ranges more closely than current methods can it be seriously considered for implementation.

(c) Integration. A study of the role of the proposed aids in the existent (or planned) combat control setting is necessary, including hardware, software, command hierarchy, and training. Integration of the aids within the combat control center requires consideration of modes of display and interaction, the appropriate personnel for operation of the aid, and ability of users to acquire through training an adequate intuitive grasp of the principles of operation of the aid.

(d) Technical refinement. Further technical study of the aids might yield improvements. In particular, the representation of interdependencies as shared information (for the purposes of subjective assessment) needs further exploration. Another issue is whether the credibility of an estimate of expected range should be assessed qualitatively (e.g., on a scale from poor to excellent), rather than being expressed as a credible interval. The relation of the pooling technique to the Time/Range plot bears further exploration, as well.

As argued in the introduction, the real pay-off of the aids is in the support they give to decision making. Thus, the present inference and alerting aids might be supplemented by aids which suggest actions (e.g., time to launch weapons or how to improve the accuracy of the solution). For example, an aid which suggests the appropriate time to shoot would weigh the risks of firing too soon against those of firing too late. It should be stressed, however, that the aid merely provides a suggestion and that the actual decision is made by the Commanding Officer.

In general, a careful study of current practices and requirements in a decision-making context is necessary before an aid can be confidently designed. Otherwise, aids may be unuseable within cognitive and organizational constraints; and even if useable, they may not be worth using if they are directed at the wrong problem. Recommendations for additional aids as well as further refinement of the currently proposed aids should be guided by research with these principles in mind.

APPENDIX A

SUBMARINE DECISION/ASSESSMENT CONTEXTS

The following list of decision and assessment contexts, while by no means exhaustive, is intended to cover a range of situations where potential room for improvement exists. Each context is characterized in terms of the objectives sought in making the decision or assessment, the decisions or assessments themselves, and a selected subset of the factors which may be considered in making the decision or assessment.

There is no implication that in practice all the factors listed are always (or even usually) taken into account. For example, contingency plans for torpedo evasion sometimes fail to provide for constraints imposed by geography (e.g., shallow water). Recent intelligence about enemy sightings may be ignored in the process of classifying a contact. In fact, it is in the need for systematic timely integration of multiple factors that room for improvement may often be found.

DECISION/ASSESSMENT CONTEXTS

- PATROL PLAN
- COMMUNICATION OF CONTACT
- CLASSIFICATION
- TARGET SELECTION
- WEAPON SELECTION
- APPROACH
- LOCALIZATION
- FIRING POINT
- POST WEAPON LAUNCH
- REATTACK/EVASION
- TORPEDO EVASION
- TRACKING
- FLOODING

PATRAL PLAN

- OBJECTIVES
  - AVOID COUNTERDETECTION
  - MAXIMIZE DETECTION CAPABILITY
  - COVER PATROL AREA
  - COPY BROADCASTS

- DECISIONS
  - DEPTH/SPEED/COURSE VS. TIME
  - TIMING OF INTENSIVE SEARCHES
  - TIMING OF PD OPERATIONS

- FACTORS
  - ENVIRONMENT (SVP, OCEAN DEPTH, ETC.)
  - SIZE/GEOGRAPHY OF PATROL AREA
  - LIKELY TARGET TYPES

COMMUNICATION OF CONTACT

- OBJECTIVES
  - TRANSMIT INFORMATION ABOUT CONTACT
  - MAINTAIN CONTACT
  - AVOID COUNTERDETECTION
- DECISIONS
  - ASCEND TO PD OR USE SONABOUYS
  - TIMING OF PD OPERATIONS
  - TYPE OF RECEPTION/TRANSMISSION AT PD
- FACTORS
  - CLASSIFICATION OF CONTACT
  - VALUE/CAPABILITIES OF CONTACT
  - RANGE, COURSE, SPEED OF CONTACT

CLASSIFICATION

- OBJECTIVES
  - AID DECISIONS ON TARGET SELECTION, ATTACK, EVASION, TRACKING, COMMUNICATION, ETC.
- ASSESSMENTS
  - TARGET CLASSIFICATION (SIDE/SIZE/TYPE/CLASS/SHIP)
  - CONFIDENCE IN POSSIBLE CLASSIFICATIONS
- FACTORS
  - SENSOR DATA
  - PRIOR RESEARCH
  - RECENT INTELLIGENCE

TARGET SELECTION

- OBJECTIVES
  - ENGAGE/TRACK MISSION-DESIGNATED TARGETS
  - ENGAGE/TRACK HIGH PRIORITY TARGETS
  - AVOID COUNTERDETECTION/COUNTERATTACK
- DECISION
  - ENGAGE/TRACK TARGET
- FACTORS
  - CLASSIFICATION OF TARGETS
  - VALUES/CAPABILITIES OF TARGETS

WEAPON SELECTION

- OBJECTIVES
  - DESTROY TARGET (REQUIRES ACCEPTABLE SEARCH CAPABILITY, MAXIMUM RANGE, MINIMUM RANGE, KILL RADIUS, DELIVERY TIME, DESTRUCTIVE FORCE)
  - AVOID COUNTERDETECTION
  - MAINTAIN WEAPON RESERVE

- DECISIONS
  - WEAPONS MIX IN TUBES
  - WEAPONS USE (TOMAHAWK/HARPOON/SUBROC/MK 48/MK 37)

- FACTORS
  - CLASSIFICATION OF TARGET
  - RANGE OF TARGET
  - TARGET ALONE OR ACCOMPANIED
  - VALUE/CAPABILITY OF TARGET

APPROACH

- OBJECTIVES
  - AVOID COLLISION
  - AVOID COUNTERDETECTION
  - MAINTAIN CONTACT
  - BRING WITHIN WEAPON RANGE
  - OBTAIN ADEQUATE TMA SOLUTION

- DECISIONS
  - APPROACH MANEUVERS
  - SOLUTION MANEUVERS
    (COURSE/SPEED/DEPTH/ASPECT VS. TIME)

- FACTORS
  - RANGE, COURSE, SPEED, DEPTH, ASPECT OF TARGET
  - CLASSIFICATION OF TARGET
  - CAPABILITIES OF TARGET
  - RANGE OF SELECTED WEAPON

<u>LOCALIZATION</u>

- OBJECTIVES
  - AID DECISIONS ON APPROACH MANEUVERS, TIME OF FIRE, TRACKING, ETC.
- ASSESSMENTS
  - TARGET RANGE/COURSE/SPEED
  - CONFIDENCE IN SOLUTION
- FACTORS
  - SENSOR DATA
  - RECENT INTELLIGENCE
  - PRIOR RESEARCH (INCL. RANGING ALGORITHMS)

FIRING POINT

- OBJECTIVES
  - KEEP INITIATIVE (FIRE BEFORE COUNTERDETECTION OR CHANGE IN TARGET STATUS)
  - MAXIMIZE CHANCE OF HIT (FIRE AFTER CLOSING WITHIN WEAPON RANGE AND OBTAINING ADEQUATE SOLUTION)

- DECISIONS
  - TIME OF FIRE

- FACTORS
  - APPROACH MANEUVERS SELECTED
  - RANGE OF TARGET
  - TMA SOLUTION ADEQUACY
  - CLASSIFICATION OF TARGET
  - VALUE/CAPABILITIES OF TARGET

POST WEAPON LAUNCH

- OBJECTIVES
  - COMPENSATE FOR TARGET MANEUVER
  - CORRECT ERRONEOUS TMA SOLUTION
  - COVER TARGET VOLUME OF UNCERTAINTY
  - INFLICT LETHAL DAMAGE

- DECISIONS
  - WEAPON GUIDANCE
  - USE OF BACKUP WEAPON

- FACTORS
  - TORPEDO MASKING POST-LAUNCH TMA
  - TORPEDO ALERTING TARGET
  - ADEQUACY OF PRE-LAUNCH TMA
  - CLASSIFICATION (SIZE) OF TARGET
  - MUTUAL INTERFERENCE BY TORPEDOES

REATTACK/EVASION

- OBJECTIVES
  - DESTROY TARGET
  - EVADE COUNTERATTACK
- DECISIONS
  - IMMEDIATE REATTACK VS. DISENGAGE, REATTACK LATER VS. DISENGAGE PERMANENTLY
- FACTORS
  - CLASSIFICATION OF TARGET
  - VALUE/CAPABILITIES OF TARGET

TORPEDO EVASION

- OBJECTIVES
  - IMMEDIATE EVASION
- DECISIONS
  - O/S COURSE/SPEED/DEPTH VS. TIME
  - USE OF DECOYS/BEACONS
- FACTORS
  - TORPEDO COURSE/SPEED/DEPTH/TYPE
  - NUMBER OF TORPEDOES
  - SOURCE OF TORPEDO
  - GEOGRAPHY

TRACKING

- OBJECTIVES
  - AVOID COLLISION
  - AVOID COUNTERDETECTION
  - MAINTAIN CONTACT
  - KEEP WITHIN RELEVANT SENSOR RANGE
  - OBTAIN ADEQUATE TMA SOLUTION
  - OBTAIN REQUIRED INFORMATION
- DECISIONS
  - RANGE/COURSE/SPEED/DEPTH/ASPECT VS. TIME
  - SENSING MODE
  - MAST EXPOSURE DURATION/EXTENT
- FACTORS
  - CLASSIFICATION OF TARGET
  - VALUE/CAPABILITIES OF TARGET
  - RANGE, COURSE, SPEED, DEPTH, ASPECT OF TARGET

<u>FLOODING</u>

- OBJECTIVES
  - CONTROL CASUALTY
  - RESTORE SHIP TO NORMALCY
- DECISIONS
  - SPEED
  - UP ANGLE
  - BLOW MAIN BALLAST
- FACTORS
  - LOCATION OF FLOODING
  - SIZE OF HOLE
  - DURATION OF FLOODING
  - THREAT TO POWER SUPPLY
  - GEOGRAPHY

APPENDIX B

MATHEMATICAL FORMULAE FOR DEA

The mean and variance of any differentiable function of random variables can be approximated from the means, variances and covariances of those variables as follows:

Let $y = F(\tilde{x}_1, \ldots \tilde{x}_n)$, then

(1) $$E(y) \simeq F(E(\tilde{x}_1), \ldots E(\tilde{x}_n)) + \tfrac{1}{2}\Sigma V(\tilde{x}_i)\, \partial^2 F/\partial E(\tilde{x}_i)^2 + \tfrac{1}{2}\sum_{i\neq j} \mathrm{Cov}(\tilde{x}_i, \tilde{x}_j)\ \partial^2 F/\partial E(\tilde{x}_i)\partial E(\tilde{x}_j),$$

(2) $$V(y) \simeq \Sigma V(\tilde{x}_i)(\partial F/\partial E(\tilde{x}_i))^2 + \sum_{i\neq j}\mathrm{Cov}(\tilde{x}_i, \tilde{x}_j)(\partial F/\partial E(\tilde{x}_i))(\partial F/\partial E(\tilde{x}_j))$$

where $E(y)$ = expectation of y, $V(y)$ = variance of y. The derivation of these approximations, from a Taylor series expansion of the function F, may be found in Brown (1971, Appendix II).

For the application of formulae (1) and (2) to Ekelund ranging, we start with

(3) $$R_T = \left(\frac{Sx_1 - Sx_2}{\dot{B}_1 - \dot{B}_2}\right) 1934 + r$$

where r represents residual error, and Sx_i and $\dot{B}_i$ are speed across line of sight and bearing rate respectively on leg i. It follows from (1) and (3) that

(4) $$E(R_T) \simeq 1934 \left[\frac{E(Sx_1 - Sx_2)}{E\ (\dot{B}_1 - \dot{B}_2)} + \frac{V(\dot{B}_1 - \dot{B}_2)E(Sx_1 - Sx_2)}{2\ (E(\dot{B}_1 - \dot{B}_2))^3} - \frac{\mathrm{Cov}(Sx_1 - Sx_2,\ \dot{B}_1 - \dot{B}_2)}{(E(\dot{B}_1 - \dot{B}_2))^2}\right] + E(r)\ ,$$

and it follows from (2) and (3) that

$$(5)\quad V(R_T) = 1934^2 \left[\frac{V(Sx_1-Sx_2)}{(E(\dot{B}_1-\dot{B}_2))^2} + \frac{V(B_1-B_2)(E(Sx_1-Sx_2))^2}{(E(\dot{B}_1-\dot{B}_2))^4} - \frac{2\cdot Cov(Sx_1-Sx_2, \dot{B}_1-\dot{B}_2)\ E(Sx_1-Sx_2)}{(E(\dot{B}_1-\dot{B}_2))^3}\right] + V(r)$$

where we assume that residual error is independent of errors in the primary readings.

Some of the expressions in (4) and (5) are further decomposed as follows:

$$(6)\quad E(Sx_1-Sx_2) = E(Sx_1) - E(Sx_2)\ ,$$

$$(7)\quad V(Sx_1-Sx_2) = V(Sx_1) + V(Sx_2) - 2\cdot Cov(Sx_1,Sx_2).$$

Similarly,

$$(8)\quad E(\dot{B}_1-\dot{B}_2) = E(\dot{B}_1) - E(\dot{B}_2),$$

$$(9)\quad V(\dot{B}_1-\dot{B}_2) = V(\dot{B}_1) + V(\dot{B}_2) - 2\cdot Cov(\dot{B}_1,\dot{B}_2).$$

When formulae (6) through (9) are substituted into (4) and (5), we have expressions for $E(R_T)$ and $V(R_T)$ very nearly in terms of the inputs specified in Figure 3-3. We need only the following additional steps:

The true value of Sx (or $\dot{B}$) is treated as the sum of the primary reading, which is known, and a variable error, δ. Therefore, by (1),

$$(10)\quad E(Sx) = \text{Prim. Reading for } Sx + E(\delta_{Sx}),$$

where $E(\hat{\delta}_{Sx})$ is a correction for the expected bias, if any, in measuring S_x. Since the primary reading is regarded as a constant, by (2) we have:

$$(11) \quad V(S_x) = V(\hat{\delta}_{Sx}).$$

(Similarly for $E(\dot{B})$ and $V(\dot{B})$.)

The covariance between x and y is calculated from the regression coefficient of x on y as follows:

$$(12) \quad Cov(x,y) = B(y|x)\ V(x).$$

Finally, the 95% credible interval for a random variable x is related to the variance of x by the following formula:

$$(13) \quad CI_{.95}(x) = 1.96 \cdot (V(x))^{\frac{1}{2}},$$

which assumes that x is normally distributed.

Somewhat more elaborate procedures are required in place of equation (13) if the assumption of normality proves to be implausible. It is likely, for example, that normality will be a better approximation for some variables after a transformation of scale.

Suppose that a random variable x is normally distributed under a continuous monotonic transformation represented by T. Let y = T(x); then

$$(14) \quad V(y) = (dT/dE(x))^2 V(x)$$

from formula (2);

$$(15) \quad CI_{.95}(y) = 1.96 \cdot V(y)^{\frac{1}{2}}; \text{ and}$$

$$\text{(16)} \quad CI^{+}_{.95}(x) = T^{-1}\,(E(y) + CI_{.95}(y)) - E\,(x)$$

$$CI^{-}_{.95}(x) = E(x) - T^{-1}\,(E(y) - CI_{.95}(y)).$$

where

$$\text{(17)} \quad E(y) = T(E(x)) + \tfrac{1}{2}\,V(x)\,d^2T/dE(x)^2$$

from formula (1). Thus, given V(x), we can derive intervals of uncertainty for x.

Conversely, given intervals of uncertainty for x, we let

$$\text{(18)} \quad CI_{.95}(y) = \tfrac{1}{2}(T(E(x) + CI^{+}_{.95}(x)) - T(E(x) - CI^{-}_{.95}(x)))$$

and solve for V(x) using equations (14) and (15):

$$\text{(19)} \quad V(x) = \left[\frac{CI_{.95}(y)}{1.96(dT/dE(x))}\right]^2 .$$

Note the motivation for these derivations. Equations (7) and (9) call for variances of errors as inputs to the DEA algorithm, and these variances can be extracted directly from previously recorded ranging data in a manner described in Appendix E. Equation (2), moreover, produces a variance on target range as its output. However, intervals of uncertainty in the original scale (e.g., range or bearing rate) are more readily comprehensible to users and so constitute a more appropriate display format. We thus need a procedure for going from variances in the original scale to intervals of uncertainty in that scale--so that inputs from prior research can be adjusted by direct judgment, and so that range uncertainty can be understood in an intuitive spatial manner. And we need to reverse that procedure so that the results of direct judgment can be used as inputs to the algorithm.

Updating. As noted in Section 3.4.6 of the text, a further application of DEA concerns updating. The range estimates from different TMA techniques may refer to different points in time. Pooling requires, on the other hand, that all range estimates refer to some common time t. Estimates of course and speed, themselves uncertain, may be used to derive a range estimate for time t for a given technique, together with a credible interval which takes account of the additional uncertainty. The following is a simplified account of how this might be done.

We refer to target range at time t as R_t. This can be expressed in terms of range at a previous time t' plus the change in range ($\Delta R_{t-t'}$). $\Delta R_{t-t'}$ is further decomposed into components due to own ship ($\Delta R_{t-t'}(O)$) and the Target ($\Delta R_{t-t'}(T)$):

$$
\begin{aligned}
(20) \qquad R_t &= R_{t'} + \Delta R_{t-t'} \\
\Delta R_{t-t'} &= \Delta R_{t-t'}(T) + \Delta R_{t-t'}(O) \\
&= \pm (t-t')S_T \cos(By-C_T) \pm (t-t')S_O \cos(C_O-By)
\end{aligned}
$$

where S_T and S_O are target and own ship speed, respectively; By is target bearing measured clockwise from North to the line of sight; C_T is target course measured from North to the target track; and C_O is own ship course measured from North to own ship track.

To simplify the formulae, we assume that the dominating sources of uncertainty in (20) are $R_{t'}$, C_T, and S_T, ignoring errors in By, S_O, and C_O. We also assume that errors in each of these three variables are independent of errors in the others. Then, applying formula (1), we get:

$$(21)\quad E(R_t) = E(R_{t'}) \pm (t-t')E(S_t)\,(1 \pm V(C_T)/2)\ \cos\ (By - E(C_T)) \pm (t-t')S_O \cos(C_O - By).$$

Applying formula (2), we get:

$$(22)\quad V(R_t) = V(R_{t'}) + V(S_T)(t-t')^2 \cos^2\ (By - E(C_T)) + V(C_T)\ (t-t')^2 \sin^2\ (By - (E(C_T)))$$

Note that updating by means of DEA differs in two respects from dead-reckoning on the basis of target course and speed (a direct application of equation 20):

(i) The updated estimate of range contains an adjustment due to possible error in the assessment of target course (i.e., $V(C_T)/2$ in equation 21).

(ii) An explicit assessment of error in the updated range estimate is also provided. This error increases with the time since the original range estimate (t-t') and is a function of the uncertainty in the target course and speed estimates used for updating (equation 22).

APPENDIX C

MATHEMATICAL FORMULAE FOR POOLING

Let E_1 be the estimate of target range produced by one technique and E_2 the estimate produced by a different technique. Typically, the values of E_1 and E_2 are not identical. The true range R may be expressed as the sum of each range estimate and an error term:

$$R = E_1 + \varepsilon_1$$

$$R = E_2 + \varepsilon_2$$

We recall from Appendix B that each ranging technique provides not only an expected target range (E_i), but also a measure (V_i) of the variance of the true range around the estimate:

$$V_i(R|E_i) = V_i(\varepsilon_i|E_i) = V_i\ (\varepsilon_i),$$

assuming independence of E_i. Let ρ be the correlation between errors in the two techniques,

$$\rho = COR(E_1, E_2|R) = COR(\varepsilon_1, \varepsilon_2),$$

assuming constancy across values of R. Then (with further assumptions to be spelled out shortly), E_1 and E_2 can be pooled by the following formula:

$$(1)\qquad E\ (R|\underline{E},\underline{V}) = \frac{\left(\frac{1}{V_1} - \frac{\rho}{\sqrt{V_1V_2}}\right) E_1 + \left(\frac{1}{V_2} - \frac{\rho}{\sqrt{V_1V_2}}\right) E_2}{\frac{1}{V_1} + \frac{1}{V_2} - \frac{2\rho}{V_1V_2}}$$

The variance of the true range around this estimate is

$$(2)\qquad V\ (R|\underline{E},\underline{V}) = \frac{1 - \rho^2}{\frac{1}{V_1} + \frac{1}{V_2} - \frac{2\rho}{\sqrt{V_1V_2}}}$$

Formula (1) is a weighted average of the range estimates, where the weight for each solution includes a term ($1/V_i$) corresponding to the assessment by that technique of its own credibility. Clearly, formula (1) is invalid unless uncertainty within the various solutions is evaluated in a consistent manner. Otherwise, for example, a range technique which tended to overstate its accuracy would exert a disproportionate influence on the pooled estimate. Consistency is imposed by the application of DEA to actual target ranging data, as described in Appendix E. As a result, the probabilities produced by each technique are calibrated: for each technique, the true range should fall outside the 95% credible interval 5% of the time. (Note that from the personalist point of view, consistency is not threatened but preserved by allowing the CO to adjust these intervals. He should do so when he feels that the current situation is not similar in respect of probability to those in which empirical data were collected (de Finetti, 1964).)

The simultaneous consideration of two (or more) solutions raises the special problem of joint calibration. The weights in formula (1) also contain a term, $\rho/\sqrt{V_1V_2}$, which (in effect) adjusts the credible intervals to reflect information about how errors in the two techniques covary. To see this, note that we could proceed as if solution errors were independent ($\rho' = 0$) with variances V_i' where

$$V_i' = \left(\frac{1}{V_i} - \frac{\rho}{\sqrt{V_1V_2}}\right)^{-1}$$

$$= V_i\left(\frac{V_j}{V_j - \rho\sqrt{V_1V_2}}\right) \text{for } i=1, j=2 \text{ and } i=2, j=1.$$

When credible intervals are based on V_1' and V_2', the true range should simultaneously fall outside <u>both</u> 95% credible intervals 0.25% (=5% x 5%) of the time. ρ, like the V_i, is assessed by reference to actual data, subject to the CO's judgment.

Jointly calibrated ranging techniques, although probabilistically consistent, will often produce non-identical range estimates. They will occasionally produce non-overlapping 95% credible intervals. Clearly, pooling is still necessary. The method represented by equations (1) and (2) draws justification from three sources: Bayesian inference; least squares; and an intuitive notion of information.

Bayesian Inference

Within the Bayesian framework, the results of the various ranging techniques are regarded as evidence, and the pooled value is the CO's inference based on his assessment of the diagnostic value of each technique. Suppose that technique i provides a probability function $f_i(R)$ on target range, with mean E_i and variance V_i. Let $F(\cdot)$ in general denote probability distributions ascribed to by the CO. $F(R|d)$ is the CO's assessment of range based on his knowledge (d) prior to receiving input from any ranging technique. Then, according to Bayes' theorem:

$$(3) \qquad F(R|f_1,\ldots,f_n,d) = k\cdot F(f_1,\ldots f_n|R,d)\cdot F(R|d)$$

where k is a normalization constant (Lindley, Tversky, Brown, 1977).

Note that formula (3), while treating the f_i as events subject to the CO's probability assessments, does not use them directly as probabilities. The CO is called upon to make a quite demanding set of second-order assessments regarding the likelihood of obtaining particular combinations of solutions given various true values of target range. However, if certain conditions are satisfied, the task is much simplified. In particular, we shall see that if the f_i are consistently calibrated, second-order assessments can be avoided.

Since we assume that the f_i are normal and therefore fully defined by the vector of means ($\underline{E}$) and variances ($\underline{V}$), we can simplify the likelihood expression in (3):

$$(4) \qquad F(f_1, \ldots, f_n \mid R, d) = F(\underline{E}, \underline{V} \mid R, d)$$

$$= F(\underline{E} \mid \underline{V}, R, d) \cdot F(\underline{V} \mid R, d)$$

and if the V_i are invariant with true range,

$$= c \cdot F(\underline{E} \mid \underline{V}, R, d).$$

where c is a constant for fixed d.

In place of (3), we now have

$$(5) \qquad F(R \mid \underline{E}, \underline{V}, d) = k \cdot F(\underline{E} \mid \underline{V}, R, d) \cdot F(R \mid d)$$

(cf., Morris, 1977).

If the E_i are normally distributed unbiased estimates of R and if $F(R|d)$ is also normal, the posterior probability $F(R|\underline{E},\underline{V},d)$ is normal with parameters which are weighted averages of the prior and likelihood parameters. We assume that the variances of the E_i are independent of the true value, R. Thus,

$$(6) \qquad V(E_i \mid V_i, R, d) = V(E_i \mid V_i, d) = \emptyset(V_i)$$

for some function $\emptyset$ independent of R.

$\emptyset(V_i)$ is the CO's assessment of the credibility of solution i taken by itself. It is the variance of the estimate E_i around the true range R. V_i, on the other hand, is the assessment by the technique itself of the variance of R around E_i.

If the CO's prior knowledge of range is relatively uncertain, $F(R|d)$ approximates a diffuse distribution. Then the posterior expected value of R, $E(R|\underline{E},\underline{V},d)$, is a weighted average of the E_i. For two solutions, E_1 and E_2,

$$(7) \qquad E(R \mid \underline{E}, \underline{V}, d) = \frac{\left(\frac{1}{\emptyset(V_1)} - \frac{\rho}{\sqrt{\emptyset(V_1)\emptyset(V_2)}}\right)E_1 + \left(\frac{1}{\emptyset(V_2)} - \frac{\rho}{\sqrt{\emptyset(V_1)\emptyset(V_2)}}\right)E_2}{\frac{1}{\emptyset(V_1)} + \frac{1}{\emptyset(V_2)} - \frac{2\rho}{\sqrt{\emptyset(V_1)\emptyset(V_2)}}}$$

In essence, equation (7) requires only three things from the CO: a judgment that the E_i are unbiased estimates of R, an assessment of the variance $\emptyset(V_i)$ of $F(E_i|V_i,R,d)$ for each technique i, and an assessment of ρ, viz. $COR(E_i,E_j|R)$, for each pair of techniques i,j. ρ can be estimated directly from empirical data (Appendix E) and subjectively adjusted in a manner to be described later in this section. Moreover, it can be shown that if the f_i are calibrated, the second-order means and variances can be derived directly from the f_i. In particular,

$$E(E_i|V_i, R,d) = R$$

i.e., the E_i are unbiased estimates of R, and

$$\emptyset(V_i) = V_i.$$

We now give a proof of the latter equality. (The proof of the former is parallel).

First note that since $\emptyset(V_i)$ is independent of R, the expected value of $\emptyset(V_i)$ with respect to R is $\emptyset(V_i)$:

$$(8) \int_R F(R|V_i,d)\ \emptyset\ (V_i)\ dR = \emptyset\ (V_i) \int_R F(R|V_i,d)dR = \emptyset(V_i)$$

Thus, by (6), (8), and the definition of variance,

$$(9) \quad \emptyset(V_i) = \int_R F(R|V_i,d)\emptyset(V_i)dR$$

$$= \int_R F(R|V_i,D) \int_{E_i} F(E_i|V_i,R,d)(E_i - R)^2\ dE_i dR$$

$$= \int_R \int_E F(R\&E_i)|V_i,d)\ (E_i-R)^2 dE_i dR.$$

Turning now to Vi, by the definition of variance

$$(10) \quad V_i = \int_R f_i(R)\ (R-E_i)^2\ dR.$$

If the f_i are calibrated, the CO can take them directly as his own probabilities:

$$(11) \quad f_i(R) = F(R|f_i,d) = F(R|E_i,V_i,d),$$

again assuming f_i is fully defined by its mean and variance.

Combining (10) and (11),

$$(12) \quad V_i = \int_R F(R|E_i,V_i,d) \ (R-E_i)^2 \ dR.$$

If V_i is independent of the range estimate E_i, the expected value of V_i with respect to E_i is equal to V_i:

$$(13) \quad \int_{E_i} F(E_i|V_i,d) \ V_i dE_i = V_i \int_{E_i} F(E_i|V_i,d) \ dE_i = V_i.$$

Thus, by (12) and (13)

$$(14) \quad V_i = \int_{E_i} F(E_i|V_i,d) V_i dE_i$$

$$= \int_{E_i} F(E_i|V_i,d) \int_R F(R|E_i,V_i,d) \ (R-E_i)^2 \ dRdE_i$$

$$= \int_{E_i} \int_R F(R\&E_i)|V_i,d) \ (R-E_i)^2 dRdE_i.$$

(9) and (14) imply that

$$(15) \quad \emptyset \ (V_i) = V_i$$

Equation (1), of course, follows from (7) and (15). (See Morris, 1977, for a stronger conclusion based on more difficult mathematics).

<u>Least Squares</u>

A quite different line of justification for the proposed pooling procedure is that it provides a least squares estimate of target range. That is, given that target range is to be estimated by a weighted average:

$$R_T = w_1 E_1 + w_2 E_2$$

with $\qquad w_2 = (1 - w_1),$

the weights in formula (1) minimize the variance of the range estimate around the true range.

It can be shown from equation (2) that the variance of the reconciled estimate is always less than or equal to the smaller of the two variances of the original estimates. There are only two cases of equality: when one technique is already perfect (has zero variance), and when

$$\rho/\sqrt{V_1V_2} = \frac{1}{V_1} \text{ or } \frac{1}{V_2}.$$ Ordinarily, therefore, pooling

results in an increase in precision. Bunn (1978) and Reinmuth and Geurts (1979) cite pertinent empirical data from the pooling of forecasts. (It remains, of course, to test this prediction with real data in the current application.)

Information

There is a natural heuristic interpretation of the weights in formula (1), in terms of information (Freeling, 1980). To the extent that these estimates draw on different sources of information, we obtain more information by utilizing both estimates than by using only one. Thus, an intuitively reasonable way of weighting the two estimates is in proportion to the information unique to each. In fact, it can be shown that the weights in equation (1) satisfy this intuitive requirement. The weight for E_1 is proportional to the partial correlation of E_1 and R given E_2. Similarly, the weight for E_2 is proportional to the partial correlation between E_2 and R given E_1:

(16) $$\frac{\frac{1}{V_i} - \frac{\rho}{\sqrt{V_1V_2}}}{\frac{1}{V_1} + \frac{1}{V_2} - \frac{2\rho}{\sqrt{V_1V_2}}} = \frac{\sqrt{V(R|E_j)}}{\sqrt{V(E_i|E_j)}} \cdot COR(E_i,R|E_j)$$

for i=1, j=2, or i=2, j=1. The correlation of the true range with E_i given E_j is a measure of the additional information about range contributed by E_i when E_j is already known. Thus, equation (16) provides a third intuitive rationale for the proposed reconciliation procedure.

This view of the weights in formula (1) leads, with the help of some further assumptions, to a natural but highly approximate procedure for subjective assessment of ρ. $1/V_i$ may be viewed as a measure of the information contained in E_i taken by itself. But equation (16) suggests that E_i is weighted by a measure of the information in E_i which is not shared with E_j. This weight is proportional to $1/V_i$ reduced by $\rho/\sqrt{V_1 V_2}$. If we can assume that

(17) $0 \le \rho/\sqrt{V_1 V_2} \le \text{Min}\,(1/V_1, 1/V_2)$,

then $\rho/\sqrt{V_1 V_2}$ may be very roughly interpreted as a measure of the information shared by E_1 and E_2 (Freeling, 1980).

The CO might provide an assessment P of "the proportion of the information in E_2 which is also contained in E_1". The following formula relates P and ρ:

$$P = \frac{\rho}{\sqrt{V_1 V_2}} \Big/ \frac{1}{V_2} = \rho \frac{\sqrt{V_2}}{\sqrt{V_1}} .$$

Note that this quantity is equal to the regression coefficient of errors in E_2 on errors in E_1.

Finally, in regard to ρ, it should be noted that DEA corrects biases which vary with known factors of the environment, maneuvers, etc. Such correction of biases will eliminate many sources of correlation between errors in different techniques. Thus, interdependency has already been addressed, albeit indirectly, in the application of DEA to the individual techniques.

APPENDIX D
MATHEMATICAL FORMULAE FOR ALERTING

The probability that the target is within weapon range (R_W) is given by:

$$(1) \quad P(R_T \leq R_W) = \int_0^{\infty} P(R_T = x)\, P(R_W \geq x)\, dx.$$

where R_T is target range and R_W is own ship weapon range.

When R_W is known with certainty and R_T is normally distributed,

$$(2) \quad P(R_T \leq R_W) = \int_0^{z} (1/\sqrt{2\pi})\, e^{-x^2/2} dx \qquad \text{with}$$

$$z = \frac{R_W - \hat{R}_T}{V(\hat{R}_T)^{1/2}}$$

where $\hat{R}_T$ is the pooled range estimate (Appendix C).

When R_W is not known with certainty, (1) can be approximated by discretizing the target range distribution into intervals of length n:

$$(3) \quad P(R_T \leq R_W) \approx n \cdot \sum_{i=1}^{\infty} P(R_T = n \cdot i)\, P(R_W \geq n \cdot i).$$

$p(R_T = n \cdot i)$ and $p(R_W \geq n \cdot i)$ can be easily assessed if R_T and R_W are assumed normal.

$$P(R_T = n \cdot i) = (1/\sqrt{2\pi})\, e^{-z^2/2} \qquad \text{with}$$

$$z = \frac{n \cdot i - \hat{R}_T}{V(\hat{R}_T)^{1/2}} \quad ; \text{ and}$$

$$P(R_W \geq n \cdot i) = \int_z^{\infty} (1/\sqrt{2\pi}) e^{-x^2 \cdot 2} \, dx \qquad \text{with}$$

$$z = \frac{n \cdot i - \hat{R}_W}{V(\hat{R}_W)^{1/2}} \quad ,$$

where $\hat{R}_W$ is expected weapon range.

APPENDIX E
FEASIBILITY OF QUANTIFICATION

The feasibility of the DEA and reconciliation aids depends upon the availability of data for estimation of the appropriate inputs. One result of DSC's recent research has been to show that such quantification is indeed possible.

E.1 Quantification for DEA Aid

Figure 3-3 shows the assessments which must be obtained from prior research in order to quantify the DEA aid in its application to Ekelund ranging. They include biases and intervals of uncertainty for the primary readings and for residual error, and three covariances.

The demonstration of feasibility proceeds in three stages:

- Examination of the raw data from Rangex and other exercises,
- Derivation of primary readings from the raw data (if required),
- Computation of statistics (means, variances, covariances) on errors in the primary readings and other quantities.

All the data appearing in this appendix (Figures E-2 through E-7) are hypothetical.

E.1.1 Rangex data. Figure E-1 partially summarizes the data which are recorded from AUTEC exercises. There are three sources of data:

- Automatic records of information from land-based sensors used to reconstruct the "actual" events of the exercise,

- Automatic records of estimates within the fire control system on board ship,

- Manual records taken on board ship.

Figure E-1 shows that both "actual" and estimated values are available in a virtually continuous manner for own ship course (C_O) and speed (S_O) and for target bearing (B_Y). Moreover, estimates of target range (R_T), speed (S_T), and course (C_T) are available periodically for each of the ranging techniques in the fire control system, and can be compared with the true values as reconstructed.

On-board estimates of the same parameters are available for manual ranging techniques (e.g., Ekelund, geo plot).

E.1.2 Derivation of primary readings. Rangex records do not include the "primary readings" required for Ekelund ranging. However, speed across line of sight and bearing rate can each be calculated from the data that is given, both for actual and estimated values. Figure E-2 illustrates how speed across line of sight (Sx) can be derived from own ship speed, own ship course, and target bearing. Figure E-3 shows how bearing rate can be calculated from bearing measurements.

E.1.3 Computation of statistics. The output of the calculations just described is shown in Figure E-4: actual and estimated values for speed across line of sight and bearing rate for each leg of each maneuver. (Pairs of such legs constitute sufficient data for calculation of an Ekelund range.) The difference between the actual and estimated values is an error term upon which the appropriate statistics can be calculated, as shown.

These statistics, in turn, provide the inputs required from prior research in Figure 3-3. The mean error is a bias term;

FIGURE E-1

DATA AVAILABLE FROM RANGEX

I. Automatic Record (every 1-3 seconds):

Includes	Reconstructed ("Actual")	Estimated on Ship	
C_O	x	x	
S_O	x	x	
D_O	x		
SNR		x	
By	x	x	
D/E		x	
R_T	x	x	For: MATE
S_T	x	x	KAST
C_T	x	x	EKELUND D/E

II. Manually Recorded Logs of Solutions and Inputs (1-3 times on an approach)

E.G., Ekelund: C_O
S_O
B_t

Time/Bearing Plot with Faired Bearing Lines

FIGURE E-2

EXTRACTION OF DATA FOR

APPLICATION OF D.E.A. TO EKELUND R_T

SPEED ACROSS LINE OF SIGHT (Sx)

GIVEN:

		Actual	Estimate
Time = 17:35:29	C_O	356.7	354.8
	S_O	11.7	12.3
	By	241.9	242.0

DERIVE:

$$Sx = S_O \cdot \text{SIN}(C_O - By)$$

Estimate: 11.3 ← 12.3 SIN(354.8 - 242.0)

Actual: 10.6 ← 11.7 SIN(356.7 - 241.9)

FIGURE E-3

EXTRACTION OF DATA - CONTINUED

BEARING RATE ($\dot{B}$)

GIVEN:

	O/S Maneuver	Actual By	Estimated By
	25	242.1	241.7
Time (sec)	26	242.0	241.7
	27	242.0	241.6
	28	241.9	241.7
	29	241.9	242.0
	.	.	.
	.	.	.
	.	.	.

DERIVE:

$\dot{B}$ = SLOPE OF REGRESSION OF BEARINGS ON TIME

Estimate: -2.0

Actual: -3.2

FIGURE E-4

COMPUTATION OF STATISTICS FOR

APPLICATION OF D.E.A. TO EKELUND R_T

RELEVANT DATA		Actual	Estimated	Error(δ)
	Sx_1	-14.1	-14.8	.7
Ekelund #1	Sx_2	11.6	10.3	1.3
	$\dot{B}_1$	2.8	3.1	-.3
	$\dot{B}_2$	-3.2	-3.6	.4
. . .				
	Sx_1	15.9	15.1	.8
Ekelund #j	Sx_2	15.2	-14.3	.9
	$\dot{B}_1$	3.3	2.7	.6
	B_2	-2.4	-2.0	-.4
. . .				

COMPUTED STATISTICS

ON ERRORS

	Mean	Variance
δ_{Sx}	1.0	.1666
$\delta_{\dot{B}}$	0	.0651

		Covariance
δ_{Sx_1}	δ_{Sx_2}	-.0560
$\delta_{\dot{B}_1}$	$\delta_{\dot{B}_2}$	.0326
$\delta_{Sx_1-Sx_2}$	$\delta_{\dot{B}_1-\dot{B}_2}$	-.0093

intervals of uncertainty around the expected bias can be calculated (given distributional assumptions) from the variances of the errors.

Figure E-5 outlines how these statistics might be conditioned on variables like signal-to-noise ratio.

Figure E-6 outlines how the mean and variance for residual error are calculated. An estimate of target range is computed using the Ekelund formula, but based on the "actual" (reconstructed) values of the primary readings. This is then compared with the actual range (as measured directly by land-based sensors) to derive an error term. The mean of these errors is a residual bias attributable to deviations from the Ekelund assumptions. And the variance is due to variability in these deviations.

E.2 Quantification for Reconciliation Aid

The reconciliation algorithm requires a credible interval for each range estimate and a measure of correlation between each technique and every combination of the other techniques. These statistics can be readily calculated from Rangex data, as outlined in Figure E-7.

Actual ranges are recorded and can be compared with range estimates. The latter may be produced by the DEA aid described above or else by the direct output of the conventional ranging technique. (In the latter case, the mean of the error terms must be added to the original range estimate, to produce an unbiased estimate.) The credible interval for a range solution can be derived from the variances of the error terms. Covariances can be calculated by pooling estimates two at a time, then pooling further estimates with previously reconciled ones.

FIGURE E-5

STATISTICS CONDITIONED ON VARIABLES WHICH CAN BE ASSESSED ON SHIP

	SIGNAL-TO-NOISE RATIO					
	-5 to -2		-2 to +2		+2 to +5	
	Mean	Variance	Mean	Variance	Mean	Variance
δ_{Sx}	.9	.2103	1.0	.1653	1.1	.1242
$\delta_{\dot{B}}$	0	.0115	0	.0732	0	.1106
	Covariance		Covariance		Covariance	
$\delta_{Sx_1}\ \delta_{Sx_2}$	-.0331		-.0602		-.0751	
$\delta_{\dot{B}_1}\ \delta_{\dot{B}_2}$	.0038		.0335		.0741	
$\delta_{Sx_1-Sx_2}\ \delta_{\dot{B}_1-\dot{B}_2}$	-.0082		-.0093		-.0104	

OTHER CANDIDATE CONDITIONING VARIABLES:

- GEOMETRY (DO OR DO NOT MANEUVER ACROSS LINE OF SIGHT
- ENVIRONMENT (SOUND VELOCITY PROFILE)
- MAGNITUDE OF QUANTITIES (BEARING RATE, SPEED, RANGE)

FIGURE E-6
COMPUTATION OF RESIDUAL
ERROR FOR EKELUND R_T

RELEVANT DATA		Actual	Calculated From Actual Components	Error(ε)
Ekelund #1	R_T	23000	21750	1250
. . .				
Ekelund #2	R_T	16000	17030	-1030
. . .				

COMPUTE STATISTICS		Mean	Variance
	ε	0	585,693

FIGURE E-7

COMPUTATION OF STATISTICS FOR RECONCILING RANGE ESTIMATES

RANGEX DATA

	R_t	Actual	Estimate (DEA or Direct)	Error
Time #1	EKELUND	23000	22055	945
	D/E	23000	18159	4841
	MATE	23000	23010	-10
	KAST	23000	15010	790
. . .				
Time #2	EKELUND	16000	17380	-1380
	D/E	16000	16300	-300
	MATE	16000	16700	-700
	KAST	16000	15090	910

COMPUTE STATISTICS

ON ERRORS

	Mean	Variance	A	B	Covariance
EKELUND	0	1,637,355	EKELUND	D/E	916,077
D/E	0	2,050,127	EKE/D/E	MATE	335,885
MATE	0	895,387	EKE/D/E/MATE	KAST	23,594
KAST	0	1,893,057	↑	. . .	

(↑) RECONCILED ESTIMATES

APPENDIX F

EXTERNAL RESEARCH SOURCES

F.1 Briefings.

A crucial role in the conceptual development of the three aids has been played by feedback received in briefings. The following have received presentations on the ideas in this report.

OFFICE OF THE CHIEF OF NAVAL OPERATIONS (OP-02)

Cpt. James Van Metre
Cpt. J. J. King

NAVAL SEA SYSTEMS COMMAND

Dr. Robert Snuggs

DEFENSE ADVANCED RESEARCH PROJECTS AGENCY

Cdr. Thomas Weiner

OFFICE OF NAVAL RESEARCH

Dr. Martin Tolcott
J. R. Simpson
Cdr. Richard Pariseau

NAVAL UNDERWATER SYSTEMS CENTER (AND CONTRACTORS)

Dr. Albert Colella
Francis Spicola
Craig Gardiner
John Davis
ASEC, Inc.
David Barry

CONSULTANTS TO DECISION SCIENCE CONSORTIUM, INC.

Cdr. Richard Pariseau (on his retirement from the Navy)
Cdr. Donald Walter
Sonalysts, Inc.

F.2 Fieldwork.

In addition, extremely valuable insights into the realistic setting of ASW were obtained from the following:

- Review of videotape recordings of at-sea approach and attack exercises on board the U. S. S. Whale (courtesy of Frank Spicola and Wayne King, NUSC)

- Observation of approach and attack exercises in the MK 117 Attack Trainer at Submarine School, Groton, Ct., involving officers and crew of the U. S. S. Finback.

REFERENCES

Brown, R. V. *Research and the credibility of estimates*. Boston: Harvard University, Graduate School of Business Administration, Division of Research, 1969.

Brown, R. V., and Lindley, D. V. *Reconciling incoherent judgments* (RIJ) - *Toward principles of personal rationality* (Technical Report TR-78-8-72). McLean, VA: Decisions and Designs, Inc., July 1978.

Bunn, D. W. *The synthesis of forecasting models in decision analysis*. Basel: Birkhauser Verlag, 1978.

deFinetti, Bruno. Foresight: Its logical laws, its subjective sources. In Kyburg, H. E., and Smokler, H. E. *Studies in Subjective Probability*. New York: John Wiley & Sons, Inc., 1964.

Freeling, A. N. S. *Alternative frameworks for the reconciliation of probability assessments* (Draft Technical Report 80-4). Falls Church, VA: Decision Science Consortium, Inc., August, 1980.

Lindley, D. V., Tversky, A. and Brown, R. V. On the reconciliation of probability assessments. *Journal of the Royal Statistical Society, Series A*, 1979, *142* (2), 146-180.

Morris, Peter A. Combining expert judgments: A Bayesian approach. *Management Science*, 1977, *23* (7), 679-693.

Reinmuth, J. E., and Guerts, M. D. A multideterministic approach to forecasting. In Makridakis, S., and Wheelright, S. C., *Studies in the management sciences*, *12*, *Forecasting*. New York: North-Holland Publishing Co., 1979.

DEPARTMENT OF THE NAVY
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Code 455

OSD

CDR Paul R. Chatelier
Office of the Deputy Under Secretary
of Defense
OUSDRE (E&LS)
Pentagon, Room 3D129
Washington, D.C. 20301

CAPT John Duncan
Office of the Secretary of Defense
(C3I)
Pentagon, Room 3C200
Washington, D.C. 20301

Department of the Navy

Mr. Phillip Andrews
Naval Sea Systems Command
NAVSEA 0341
Washington, D.C. 20362

CDR Thomas Berghage
Naval Health Research Center
San Diego, CA 92152

CDR P.M. Curran
Code 604
Human Factors Engineering Division
Naval Air Development Center
Warminster, PA 18974

Commanding Officer
ONR Branch Office
ATTN: Dr. C. Davis
536 South Clark Street
Chicago, IL 60605

Commanding Officer
ONR Western Regional Office
ATTN: Dr. E. Gloye
1030 East Green Street
Pasadena, CA 91106

Commanding Officer
ONR Western Regional Office
ATTN: Mr. R. Lawson
1030 East Green Street
Pasadena, CA 91106

Department of the Navy

Commanding Officer
ONR Eastern/Centeral Regional Office
ATTN: Dr. J. Lester
Building 114, Section D
666 Summer Street
Boston, MA 02210

Commanding Officer
CAPT Richard L. Martin, USN
USS Carl Vinson (CVN-70)
Newport News Shipbuilding/Dry
Dock Company
Newport News, VA 23607

Commander
Naval Air Systems Command
Human Factors Programs
NAVAIR 340F
Washington, D.C. 20361

Commander
Naval Electronics Systems Command
Human Factors Engineering Branch
Code 4701
Washington, D.C. 20360

Dean of the Academic Departments
U.S. Naval Academy
Annapolis, MD 21402

Dean of Research Administration
Naval Postgraduate School
Monterey, CA 93940

Director
Engineering Psychology Programs
Code 455
Office of Naval Research
800 North Quincy Street
Arlington, VA 22217 (5 cys)

Director
Undersea Technology
Code 220
Office of Naval Research
800 North Quincy Street
Arlington, VA 22217

Department of the Navy

Director
Communication & Computer Technology
Code 240
Office of Naval Research
800 Quincy Street
Arlington, VA 22217

Director
Tactical Development & Evaluation
Support
Code 230
Office of Naval Research
800 Quincy Street
Arlington, VA 22217

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Office of Naval Research
800 North Quincy Street
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Code 436
Office of Naval Research
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Arlington, VA 22217

Director
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Code 437
Office of Naval Research
800 North Quincy Street
Arlington, VA 22217

Director
Naval Research Laboratory
Technical Information Division
Code 2627
Washington, D.C. 20375 (6 cys)

Department of the Navy

Dr. Jerry C. Lamb
Submarine Sonar Department
Code 3293
Naval Underwater Systems Center
New London, CT 06320

Dr. James McGrath, Code 311
Navy Personnel Research and
Development Center
San Diego, CA 92152

Dr. W. Mehuron
Office of the Chief of Naval
Operations, OP 987
Washington, D.C. 20350

Dr. George Moeller
Human Factors Engineering Branch
Submarine Medical Research Lab
Naval Submarine Base
Groton, CT 06340

Dr. Robert French
Naval Ocean Systems Center
San Diego, CA 92152

Dr. Gary Poock
Operations Research Department
Naval Postgraduate School
Monterey, CA 93940

Dr. A. L. Slafkosky
Scientific Advisor
Commandant of the Marine Corps
Code RD-1
Washington, D.C. 20380

Dr. Robert G. Smith
Office of the Chief of Naval
Operations, OP987H
Personnel Logistics Plans
Washington, D.C. 20350

Dr. Alfred F. Smode
Training Analysis and Evaluation
Group
Naval Training Equipment Center
Code N-00T
Orlando, FL 32813

Department of the Navy

Dr. Bruce Wald
Communications Sciences Division
Code 7500
Naval Research Laboratory
Washington, D.C. 20375

CDR. G. Worthington
Office of Chief of Naval
Operations OP 987
Washington, D.C. 20350

Dr. Andreas B. Rechnitzer
Office of the Chief of Naval
Operations OP 952F
Naval Oceanography Division
Washington, D.C. 20350

Mr. Ronald A. Erickson
Human Facors Branch
Code 3194
Naval Weapons Center
China Lake, CA 93555

Mr. Paul Heckman
Naval Ocean Systems Center
San Diego, CA 92152

Human Factors Department
Code N215
Naval Training Equipment Center
Orlando, FL 32813

Human Factors Engineering Branch
Code 1226
Pacific Missile Test Center
Point Mugu, CA 93042

Mr. Warren Lewis
Human Engineering Branch
Code 8231
Naval Ocean Systems Center
San Diego, CA 92152

Human Factor Engineering Branch
Naval Ship Research and Development
Center, Annapolis Division
Annapolis, MD 21402

Department of the Navy

LCDR W. Moroney
Code 55MP
Naval Postgraduate School
Monterey, CA 93940

Mr. Merlin Malehorn
Office of the Chief of Naval
Operations (OP 102)
Washington, D.C. 20350

Navy Personnel Research and
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Code 210
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Mr. Arnold Rubinstein
Naval Material Command
NAVMAT 08D22
Washington, D.C. 20360

Department of the Army

Director, Organizations and
Systems Research Laboratory
U.S. Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

Technical Director
U.S. Army Human Engineering Labs
Aberdeen Proving Ground, MD 21005

Department of the Air Force

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Maxwell Air Force Base, AL 36112

Dr. Donald A. Topmiller
Chief, Systems Engineering Branch
Human Engineering Division
USAF AMRL/HES
Wright-Patterson AFB, OH 45433

U.S. Air Force Office of Scientific
Research
Life Sciences Directorate, NL
Bolling Air Force Base
Washington, D.C. 20332

Foreign Addresses

Director, Human Factors Wing
Defence and Civil Insititute of
Environmental Medicine
Post Office Box 2000
Downsview, Ontario M3M 3B9
CANADA

Dr. A.D. Baddeley
Director, Applied Psychology Unit
Medical Research Council
15 Chaucer Road
Cambridge, CB2 2EF
ENGLAND

Dr. Kenneth Gardner
Applied Psychology Unit
Admiralty Marine Technology
Establishment
Teddington, Middlesex TW11 OLN
ENGLAND

Foreign Addresses

North East London Polytechnic
The Charles Myers Library
Livingstone Road
Stratford
London E15 2LJ
ENGLAND

Other Government Agencies

Defense Advanced Research Projects
Agency
1400 Wilson Boulevard
Arlington, VA 22209

Defense Technical Information Center
Cameron Station, Bldg. 5
Alexandria, VA 22314 (12 cys)

Dr. Judith Daly
Cybernetics Technology Office
Defense Advanced Research Projects
Agency
1400 Wilson Boulevard
Arlington, VA 22209

Dr. Craig Fields
Director, Cybernetics Technology
Office
Defense Advanced Research Projects
Agency
1400 Wilson Boulevard
Arlington, VA 22209

Professor Douglas E. Hunter
Defense Intelligence School
Washington, D.C. 20374

Other Organizations

Dr. Robert Mackie
Human Factors Research, Inc.
5775 Dawson Avenue
Goleta, CA 93017

Dr. Gary McClelland
Institute of Behavioral Sciences
University of Colorado
Boulder, Colorado 80309

Other Organizations

Dr. Miley Merkhofer
Stanford Research Institute
Decision Analysis Group
Menlo Park, CA 94025

Dr. Jesse Orlansky
Institute for Defense Analyses
400 Army-Navy Drive
Arlington, VA 22202

Dr. Arthur I. Siegel
Applied Psychological Services, Inc.
404 East Lancaster Street
Wayne, PA 19087

Dr. Paul Slovic
Decision Research
1201 Oak Street
Eugene, OR 97401

Dr. Amos Tversky
Department of Psychology
Stanford University
Stanford, CA 94305

Dr. W.S. Vaughan
Oceanautics, Inc.
422 6th Street
Annapolis, MD 21403

Dr. Gershon Weltman
Perceptronics, Inc.
6271 Variel Avenue
Woodland Hills, CA 91364

Dr. Robert Williges
Human Factors Laboratory
Virginia Polytechnical Institute
and State University
130 Whittemore Hall
Blacksburg, VA 24061

Dr. Meredith P. Crawford
American Psychological Association
Office of Educational Affairs
1200 17th Street, N.W.
Washington, D.C. 20036

Other Organizations

Dr. Ward Edwards
Director, Social Science Research
Institute
University of Southern California
Los Angeles, CA 90007

Dr. Charles Gettys
Department of Psychology
University of Oklahoma
455 West Lindsey
Norman, OK 73069

Dr. Kenneth Hammond
Institute of Behavioral Science
University of Colorado
Room 201
Boulder, Colorado 80309

Dr. James H. Howard, Jr.
Department of Psychology
Catholic University
Washington, D.C. 20064

Dr. William Howell
Department of Psychology
Rice University
Houston, Texas 77001

Dr. John Payne
Duke University
Graduate School of Business
Administration
Durham, NC 27706

Dr. Baruch Fischhoff
Decision Research
1201 Oak Street
Eugene, OR 97401

Dr. Andrew Sage
University of Virginia
School of Engineering and Applied
Science
Charlottesville, VA 22901

Human Resources Research Office
300 N. Washington Street
Alexandria, VA 22314

Other Organizations

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1200 17th Street, N.W.
Washington, D.C. 20036 (3 cys)

Professor Howard Raiffa
Graduate School of Business
Administration
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Soldiers Field Road
Boston, MA 02163

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Mr. Donald Walter
Doty Associates
416 Hungerford Drive
Suite 434
Rockville, Maryland

Capt. Frank Caldwell
Chief Staff Officer
Devron Twelve
Naval Submarine Base
New London
Groton, Connecticut 06340

Honorable Robert B. Pirie, Jr.
Assistant Secretary of Defense
(MRA & L)
Room 3-E808
Pentagon
Washington, D.C. 20301

~~Com.~~ CAPT S. L. Ward
Commander
Devron Twelve
Naval Submarine Base
New London
Groton, CT 06340

Don Hurta
Naval War College
Newport, Rhode Island 02840

Adm. Ed Peebles
Naval Sea Systems Commander
Department of the Navy
Room 7S18 NS-3
Code NAVC92
Washington, D.C. 20362

Mr. Donald Cardin
Code 01A
Naval Underwater Systems Center
Newport, Rhode Island 02840

Mr. Patrick La Rue
PME 108-T11
Naval Electronic Systems Command
Washington, D.C. 20360

Dr. Albert Colella
Combat Systems Staff
Code 3502
Naval Underwater Systems Center
Newport, Rhode Island 02840

Mr. Francis Spicola
Command and Control Branch
Code 3524
Naval Underwater Systems Center
Newport, Rhode Island 02840

Mr. John Davis
Analysis Branch
Code 3522
Naval Underwater Systems Center
Newport, Rhode Island 02840

Mr. Craig Gardiner
Submarine Exercise Programs
Code 232
Naval Underwater Systems Center
Newport, Rhode Island 02840

Mr. David Barry
FBM Systems Development Division
Code 353
Naval Underwater Systems Center
Newport, Rhode Island 02840

Capt. J. M. Van Metre
Naval Sea Systems Command
PMS-409
Washington, D.C. 20362

Capt. J. J. King
Office of the Chief of Naval Operations
Washington, D.C. 20350

Capt. J. H. Patton, Jr.
Naval Tactical Training Department
Code 10
Naval Submarine School
Groton, Connecticut 06349

Dr. Bernard McCabe
Daniel H. Wagner Associates
Station Square One
Paoli, PA 19301

Dom Carlino
ASEC, Inc.
Newport, Rhode Island 02840

Capt. Robert L. Bovey
Office of the Secretary of Defense
Washington, D.C. 20350

LCDR Kevin J. Reardon
Devron Twelve
Box 70, Naval Submarine Base
Groton, CT 06340

Mr. Thomas Downey
Analysis and Technology
P. O. Box 220
North Stonington, CT 06359